MAT 175 Deliverables Document

TABLE OF CONTENTS:

How to Navigate this Document.

a) **Find the topic of interest.**
b) **Type the page number at the top of the PDF file. Hit Enter. (Note: At the top of this document (in Adobe), there is a box that tells you the page number you are on / the total number of pages: Example 3/148, states page 3 out of 148. Delete the 3, and substitute the page number of interest).**

MAT 175 Deliverables Document

For Use as an OER Text/Guide

For Students and Instructors

MAT-175 Corequisite for Math For Liberal Arts

a. Required Topic 1a: Organization of various sets of data.

MAT-175 Required Topic 1a is intended to support the following MAT-120 topics:

1. Formal Logic: Inductive and Deductive Reasoning
2. Formal Logic: Subset, Union, Intersection and Complement
3. Probability and Statistics: Sources of potential bias within research

Khan Academy Support: https://www.khanacademy.org/math/algebra-home/alg-series-and-induction/alg-deductive-and-inductive-reasoning/v/deductive-reasoning-1

https://www.khanacademy.org/math/statistics-probability/probability-library/basic-set-ops/v/subset-strict-subset-and-superset

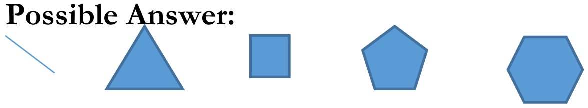
1. Formal Logic: Inductive and Deductive Reasoning

Goal: a. Demonstration of Pattern Recognition, b. Demonstration of Organization of Information

Example 1:

Organize the following objects in a reasonable way:

Possible Answer:



We observe that we organized these figures from left to right by number of sides. This isn't the only way we could have organized the objects, but they are now organized in a specific manner.

Example 2:

Organize the following data in a reasonable way:

a,d,c,b,f,g,e,h:

Possible Answer:

a,b,c,d,e,f,g,h

We put the letters in alphabetical order.

Example 3:
Name the next number in the pattern:

1,3,5,7,9,

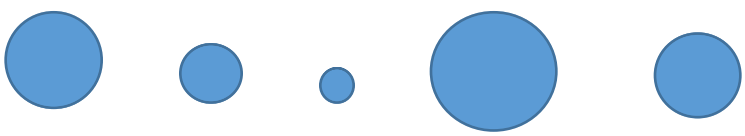
Answer: 11. We notice that these are all odd numbers, or if n is any integer greater than or equal to 1, then the pattern follows with 2n-1, with n=1,2,3,…

<u>Definitions For Inductive and Deductive Reasoning:</u>
Inductive Reasoning: The process of reasoning to a general conclusion through observations of specific cases. This involves applying pattern recognition to predict the next outcome, and then forming a general conclusion.
Deductive Reasoning: The process of reasoning to a specific conclusion from a general statement.

Exercise Set 1:
Organize the following data in a reasonable way:

a) 4,2,8,6,12,10

b) i,k,j,m,l,p,n,o

c) 

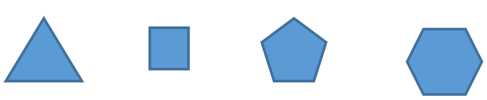
d) Chihuahua, Great Dane, Yellow Lab

e) Test Scores: 81,65,100,39,52,97,45

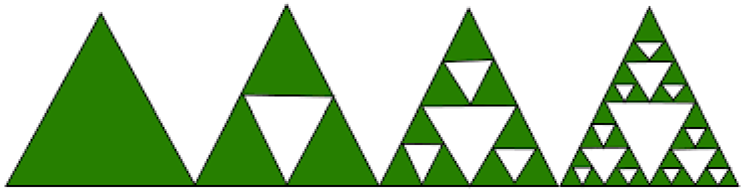
Exercise Set 2:
Recognize the pattern, and use inductive reasoning to predict the next object:

a) 1,2,3,4,5,6:

b) a,c,e,g,i:

c) 

d) . , .. , … , …. , ….. , :



e)

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 1.1
- Mathematical Excursions, Aufmann, Lockwood, Nation, Clegg, 4th Ed.
- Chapter 1.1

2. Formal Logic: Subset, Union, Intersection and Complement

Goal: To Understand How to build a Set by Organizing Data in a Meaningful Way

Definitions:

Set: A set is a collection of objects. The objects are called elements or members. A set is *well-defined* if its contents can be clearly identified.

In this section we will stick with sets in roster form. We will cover set builder notation in the mathematical notation section.

In roster form we use {} to form a set. We place all elements inside the curly brackets. Order does not matter.

Universal Set: We denote a universal set by *U*. This set contains all the elements in all the sets that pertain to a specific category or discussion of interest.

Example: Let us examine the following objects: Chair, table, bed, dresser, coffee table, couch, lamp, rug, picture, mirror, stool, curtains.

Let us create logical sets by organizing the objects.

Let *U* be the collection of some common household furniture and objects. We will place all the objects we have in the universe *U*: {chair, table, bed, dresser, coffee table, couch, lamp, rug, picture, mirror, stool, curtains}.

Let us call *A* the set of objects known to be common bedroom items: {chair, bed, dresser, lamp, rug, picture, mirror, curtains).

Let us call *B* the set of objects known to be common living/dining room items: {chair, table, coffee table, couch, lamp, rug, picture, stool, curtains}.
Note that set *A* and set *B* are all contained in set *U*. Also note that some items are in both *A* **and** in set *B*. This is known as the intersection where the two sets overlap. You will learn about this in detail in your MAT120 class.

We also note that this is not the only way we could have organized the data. We could have created more sets ,and/or organized them differently.

The Universe *U* can also contain more objects than the other combined sets created.

Exercise Set 3:
Create a Universe, *U*, along with other sets to organize the following data: (Note: There is more than one possible answer): Name each set with a letter of your choice:

a) a,b,c,i,j,k,r,s,t,x,y,z:

b) lodgepole, rose, aspen, maple, barley, cottonwood, tulip, lily, orchid, wheat, corn:

c) California, Colorado, Kansas, Florida, Oregon, Ohio, Washington, Maine, Virginia, North Carolina, Nebraska:

d) 1,2,3,4,5,6,7,8,9,10:

e) Arithmetic, Anthropology, Algebra, Music, Trigonometry, Communications, Calculus, Set Theory, Language, Geometry, Biology, Physics, Chemistry, Psychology, Art, History, Social Studies:

- Additional Textbook Support
- Mathematical Excursions, Aufmann, 4th Ed.
- Chapter 2.2

4. Probability and Statistics: Sources of potential bias within research

Goal: To recognize misuses of Statistics.

Your MAT 120 textbook has a good section on this. I don't think we need to put more examples in here. The instructor can include more examples if necessary.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8^{th} ed.
    - Chapter 1.2, Chapter 2.1, Chapter 13.2

b. Required Topic 1b: Breakdown of application (word) problems into their basic components.

MAT-175 Required Topic 1b is intended to support the following MAT-120 topics:

1. Formal Logic: Venn Diagrams
2. Modeling Real-World Financial Problem: Cost estimation using applied geometry
3. Math Modeling/Real World Applications: Direct and inverse variation

Khan Academy Support: https://www.khanacademy.org/math/pre-algebra/pre-algebra-equations-expressions/pre-algebra-alg-expression-word-problems/v/writing-basic-expressions-from-word-problems-examples

https://www.khanacademy.org/math/3rd-engage-ny/engage-3rd-module-7/3rd-module-7-topic-e/v/comparing-area-and-perimeter?modal=1

Application/Word Problems – General

We will start by addressing word problems in general before moving onto the specific topics outlined above.

General Steps for Any Application Problems.

1) Read the problem carefully.
2) Identify pertinent information. Organize this data. Discard extraneous information not needed in the problem.
3) Draw a picture if possible.
4) Label the picture and/or choose variables.
5) Write expressions and/or equations.
6) Perform any necessary substitutions.
7) Solve the equation(s).
8) Check your answer(s).

Example 1: Karen has a combination of dimes and quarters. If she has $1.80 in her purse, and she has twice and many dimes as quarters, how many does she have of each?

Let us go through each step:

1) We have carefully read through the problem.

2) There isn't really any extraneous information in this problem to discard. We recognize that number of dimes times $.10 plus number of quarters times $.25 must equal $1.80 when we are done.

3) A picture won't help much in this instance. We skip this step.

4) We will choose our variables. We let $x =$ number of quarters. We now have dimes being $2x,$ since we have twice as many.

5) We can now write an equation. We get $(.10) \cdot 2x + (.25) \cdot x = 1.80.$

6) We now solve the linear equation. First we multiply to get $.2x + .25x = 1.80.$ We now add like terms to get $.45x = 1.80.$ We divide 1.80 by $.27$ to get 4. We have 4 quarters. We have twice as many dimes, and we get 8 dimes.

7) We check our answer: $(.10) \cdot 2 \cdot 4 + (.25) \cdot 4 = 1.80.$ It worked!

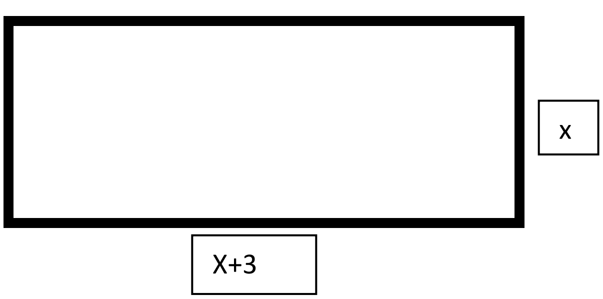
Example 2: We have a rectangle. The rectangle will be used to build a frame for some outdoor plants. We would like to be able to fit 25 plants inside of it. We know the length is 3 times the width. The total perimeter is 43 feet. What are the length of the sides?

1) We have read the problem carefully.

2) We recognize that what the rectangle will be used for and how many plants it needs is extraneous information, so we discard that info. We take note that a) It's a rectangle, b) the length is 3 times the width, and c) the perimeter is 43 feet.

3) We draw a picture.

4) We choose our variables by labeling the picture.



5) Next, we will write an equation: $x + x + 3 = 43.$

6) We will solve the linear equation: $2x + 3 = 43.$ We subtract the 3 to get $2x = 40.$ We divide each side by 2 to get $x = 20.$ This is the width. The length is $x + 3,$ which gives a length of 23.

7) We check our answer: $2 \cdot 20 + 3 = 43.$

Exercise Set 1:

Write out steps 1-5 for the following problems. (Note: you do not have to solve these problems):

1) The sum of two numbers is 12. If the first number is twice the value of the second number, what is the value of each number?

2) The sum of two numbers is 30. If the first number is three times the second number plus 2 more, what is the value of each number?

3) You have a rectangle. The length of the rectangle is twice its width. Its perimeter is 27 inches. What are the dimensions?

4) You have an isosceles triangle (recall that this means two sides are the same length). The triangle will be used to make a large picture frame. If the two sides are twice as long as the third side, and the perimeter is 25 feet, what are its dimensions?

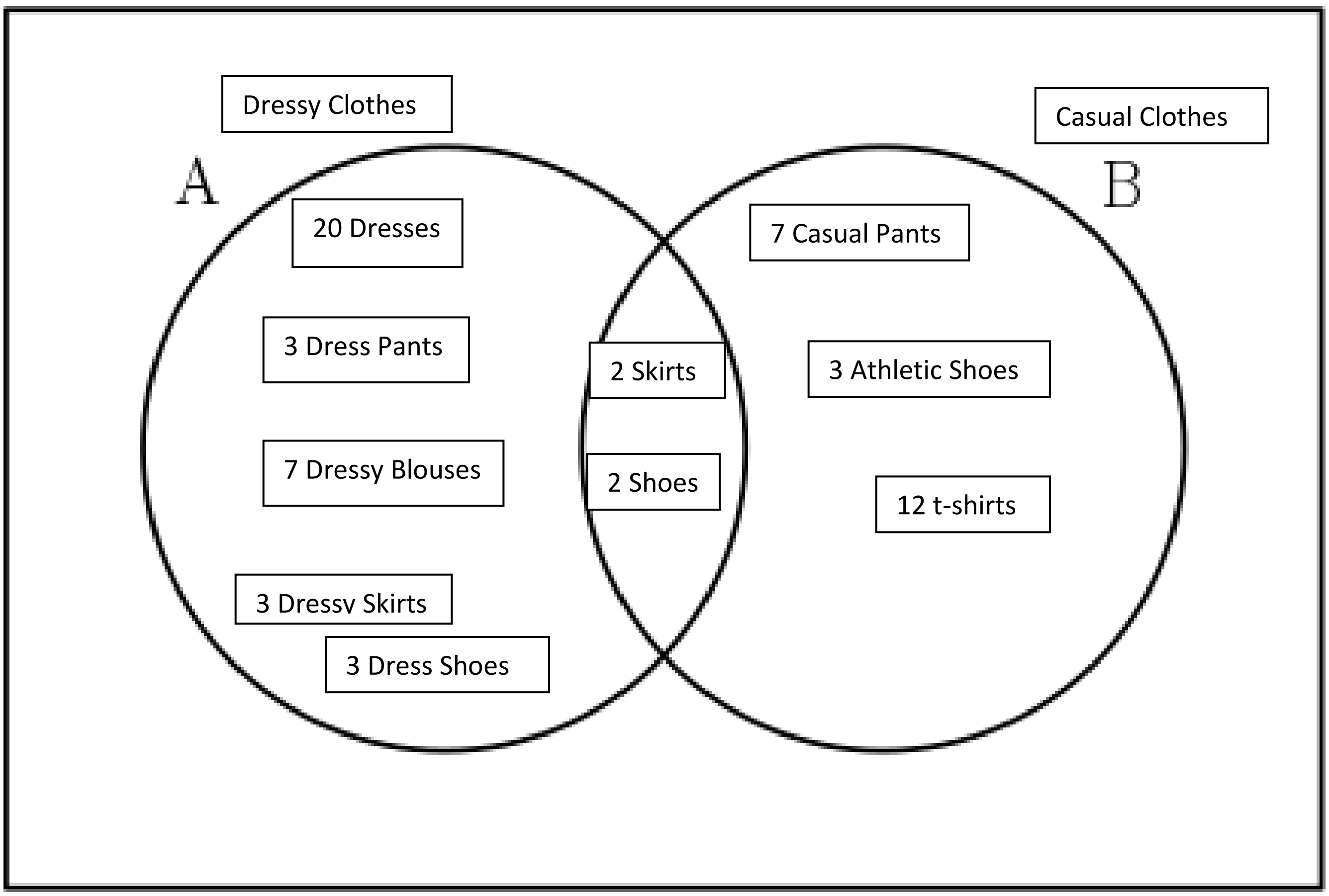
Exercise Set 2:

Complete steps 5) and 6) for all the above problems, i.e., solve and check answers.

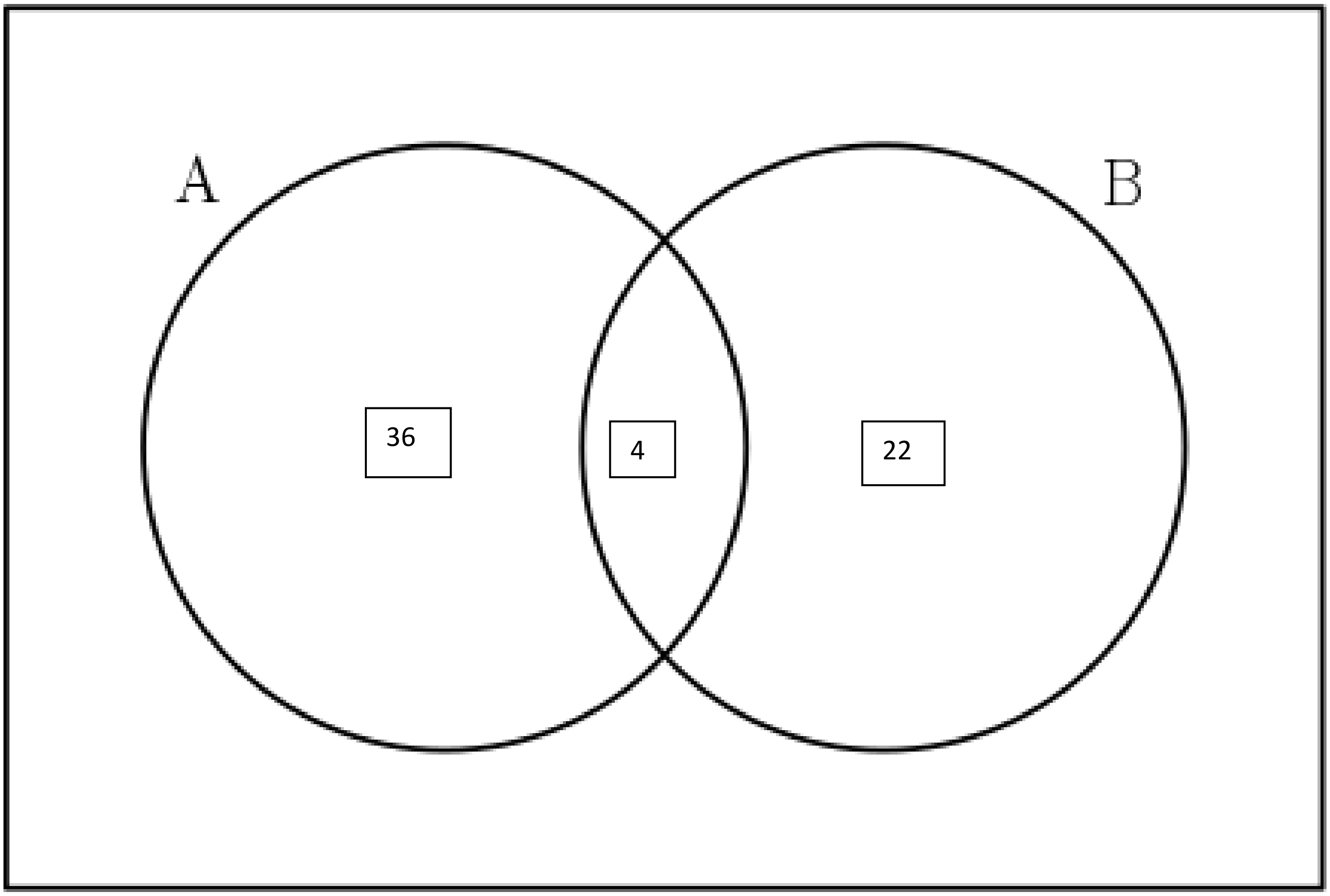
1. Formal Logic: Venn Diagrams

Goal: Mastery of simple applications for Sets using Venn Diagrams. (Note: This goal is similar to problems in MAT120, but they will be simpler in nature).

Example 1: Let us organize our closet: Mary has 20 dresses, 10 pairs of pants (3 dressy and 7 casual), 7 dressy blouses, 5 skirts (3 are dressy, 2 are both dressy and casual), 8 pairs of shoes (3 dress, 2 are dressy and casual, and 3 athletic), and 12 t-shirts. We first note that Mary has 62 items of clothing in her closet. We have decided to organize these into two groups: Dressy and Casual. We will Create a Venn Diagram.



Next we will recreate this Venn Diagram with only the numbers added in each section:



Now we can answer simple questions using the Venn Diagram:

1) How many clothes does Mary have? We add up all the numbers in all bubbles: 62.
2) How many dressy clothes does Mary have? 36+4=40 dressy clothes. The 4 in the middle overlaps with dressy and casual. These clothes are both dressy **AND** casual so they count as both.
3) How many casual clothes does Mary have? Mary has 22+4=26 by the same logic as 2).
4) How many clothes that are strictly dressy does Mary have? 36.

Exercise Set 1:

1) Construct a Venn Diagram like the second diagram in our Example 1.

Dave surveyed 25 of his friends: He found 10 liked Rock Music, 12 liked Rap Music, and 3 liked neither. He also found that 3 liked both. (Hint: Any number that falls into the neither category goes outside of the circles, but inside the rectangle: They are in the Universe, but not in Set A or Set B.)

2) Answer the following questions about question 1):

   a) How many friends liked only Rock Music?

   b) How many friends liked Rock Music or Rap Music? (hint: or means Rock, Rap or both).

3) Construct a Venn Diagram for:

George likes all the paintings in his house: 3 are only black and white, 2 are only color, and 1 is half color and half grayscale (black and white).

4) Answer the following questions about question 3):

   a) How many paintings does George have in his house?

   b) How many paintings have black and white on them?

2. Modeling Real-World Financial Problem: Cost estimation using applied geometry

Goal: Applying Geometry specifically to cost.

REVIEW THE GEOMETRY SECTION IN THE PREVIOUS GENERAL PROBLEMS.

Example 1:

For this problem we will need to also refer to Required Topic 6g. All we need for this problem is to recall the area of a rectangle is length times width.

We have a floor that we want to install carpet that is 20 feet by 30 feet. If the carpet is \$5 per square foot, how much will it cost to install?

We first find the area of the rectangle. (Note: We can follow all the steps from our General Section, but it is not really necessary for this problem as we will have no variables). Area is 20 x 30 =600 square feet. We multiply 600 Square feet by \$5 per spare foot. $600\ ft^2 \cdot \frac{\$5}{ft^2}$. Notice that the ft^2 cancel. We get 600 times \$5 to get \$3000 total.

Exercise Set 2:

1) We want to paint a wall in our house. If the wall is 15 feet by 17 feet, and the paint will cost \$10 per square foot, how much will it cost to paint the wall.

2) Make up your own problem and solve it.

3. Math Modeling/Real World Applications: Direct and inverse variation:

In this section we will just go over the formulas, and learn to write them down. We will solve problems in MAT 120.

Definitions: We have y varies directly with $x \rightarrow \quad y = kx,$ where k is the constant of proportionality.

We have y varies inversely with $x \rightarrow \quad y = \frac{k}{x}$, where k is the constant of proportionality.

Example 1: t varies directly with m. Write down the equation: $t = km$.

Example 2: x varies inversely with z. Write down the equation: $x = \frac{k}{z}$.

Example 3: t varies directly with the product of xy. Write down the equation: $t = kxy$.

Example 4: x varies inversely with the square of y. Write down the equation: $x = \frac{k}{y^2}$.

Exercise Set 3:

For the following set of exercises, write down the equation:

1) m varies directly with n.
2) x varies inversely with t.
3) t varies directly with the product of uvw.
4) u varies inversely with the cube of z.
5) w varies directly with the product of t and the square of u.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 2.1-2.4., Chapter 9.3, Chapter 6.5.
- Mathematical Excursions...
- Chapter 2.2, Chapter 7.3

c. Required Topic 1c: Study a variety of real-world applications, and find various methods for solving.

MAT-175 Required Topic 1c is intended to support the following MAT-120 topics:

1. Formal Logic: Introduction to Conjunction, Disjunction, & Negation
2 Math Modeling/Real World Applications: Application of linear models
4. Modeling Real-World Financial Problem: Annuities with applications

Khan Academy Support: **https://www.khanacademy.org/humanities/grammar/parts-of-speech-the-preposition-and-the-conjunction/introduction-to-conjunctions/v/coordinating-conjunctions-final**

https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-linear-equations-functions/8th-linear-functions-modeling/v/exploring-linear-relationships

https://www.khanacademy.org/economics-finance-domain/macroeconomics/monetary-system-topic/macroeconomics-interest-rates-and-the-time-value-of-money/v/introduction-to-present-value

1. Formal Logic: Introduction to Conjunction, Disjunction, & Negation

Goal: a. Understanding of the definitions, b. Demonstration simple applications.

Let us start with the definitions:

Compound Statement: A statement that combines two or more simple statements. They can be combined or connected with: And, Or, If→ Then, If And Only If.

Conjunction: Is a way to combine simple statements with the word **And**. We use this symbol: $\wedge$.

Disjunction: Is a way to combine simple statements with the word **Or**. We use this symbol: $\vee$.

Negation: Is a **Not (~),** statement.

Examples:

Conjunction: Sue is going to the store **and** going swimming.

Disjunction: Mark is studying math **or** he is going to eat lunch.

Negation: Jenny is not walking her dog today.

Exercise Set 1:

For the following set of exercises, identify if the compound statement is a Conjunction, Disjunction, or Negation, or a combination of more than one. If it is more than one, identify each component of the sentence.

1) John is not going to ride his bike today.
2) Sarah is going outside and she is walking, or she is working on her computer.
3) Melvin is having a good time with friends, or he is not doing his homework.
4) George is waiting outside and he is not happy.
5) Sally is looking for berries or she is hunting rabbits, and she is not having the most fun.
6) Fred is snowboarding and he is wearing a jacket, or he is not in the lodge.

Exercise Set 2:

For the following set of exercises, write 2 compound statements using a Conjunction, 2 using a Disjunction, and 2 using a Negation. Write 1 statement using a Conjunction and Negation, 1 statement using a Conjunction and Disjunction, 1 statement using a Disjunction and Negation, and 1 statement using all 3.

2. Math Modeling/Real World Applications: Application of linear models

Goal: a. Ability to Construct a Linear Model, b. Ability to Apply Basic Linear Models.

Definition: A Linear Model is a relationship between two variables with a constant rate of change (slope).

Review:

SLOPE OF A LINE: Let us first review the equation of a line. We will start with the idea of **slope**. What is a **slope**? The **slope** is a rate of change of a function. (What is a function? A function is a relationship where each input value has exactly one output value). The slope of a line is a constant rate of change. We denote slope with the symbol m. $m = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. Recall that points can be written as ordered pairs like (a, b), where a is the x-value, and b is the y-value.

Example 1: Let us find the slope between these two points. $(1,2), (3,6)$. $m = \frac{6-3}{3-1} = \frac{3}{2}$. (Note that the order does not matter as long as you are consistent with the numerator and denominator.

EQUATION OF A LINE:

Slope-Intercept Form: $y = mx + b$, where m is the slope, and b is the y-intercept.

Example 2: $y = 3x - 5$. Let us determine the slope, and the y-intercept. We note from the formula that $m = 3,\ \ b = -5$.

Example 3: Write the equation of the line, given the slope is 7, and the y-intercept is $\frac{1}{2}$.

$$y = 7x + \frac{1}{2}.$$

Point-Slope Form: Point-Slope Form is another form for the equation of a line. To use this form, we must know the slope of the line and a point. Note that if we know two points, we can find the slope.

Point-Slope Form: $y - y_1 = m(x - x_1)$.

Example 4: Find the equation of the line given the slope is 3, and a point is $(2,4)$. Let us substitute our knowns into $y_1, x_1, and\ m$.
We get $y - 4 = 3(x - 2)$. We now solve for y to put it back in slope-intercept form. First, we distribute the 3. $y - 4 = 3x - 6$. Lastly, we add the 4 to get $y = 3x - 2$.

Example 5: Find the equation of the line between the two points: Let us use the points ($(2.4), (1,6)$. First we find the slope m. $m = \frac{6-4}{1-2} = \frac{2}{-1} = -2$. Next we substitute our values into our formula: It does not matter which point we use. When we simplify, we will get the same answer either way. I choose, $(2,4)$. $y - 4 = -2(x - 2)$. We now solve for y to get our equation in slope-intercept form.

$$y = -2x + 8.$$

Exercise Set 1:

For the following set of exercises, find the slope between the 2 points:

1) (1,2), (2,3)
2) (2,4), (7,8)
3) (4,2), (9,1)
4) (1,6), (2,2)

Exercise Set 2:

For the following set of exercises, find the slope m, and the y-intercept, b. (Note: For some, you may need to put it in slope-intercept form, first).

1) $y = 2x - 8$
2) $y = \frac{1}{2}x + 7$
3) $y = \frac{2}{3}x - \frac{1}{2}$
4) $3x - y = 9$

5) $2x - 4y = 8$

Exercise Set 3:

Write the equation of the line using the appropriate formula. Write your final answer in slope-intercept form.

1) $m = 2, b = 5$
2) $m = 1, b = \frac{2}{3}$
3) $m = 9, b = \frac{1}{2}$
4) $m = 5,$ point is (1,2).
5) $m = 2,$ point is $\left(1, \frac{1}{2}\right)$.
6) Points are (1,2), (3,4)
7) Points are (2,2), (4,1).

Application of Linear Models:

Linear models are used to model real-world data. These are used when the data points available best follow a line. For example, in business and science, judgements are often based on past data. The data is then used to construct a mathematical model such as a graph, in which we can find an equation. In this case, it will be linear (or the equation of a line). The easiest example is when we have two points.

Example 6: One of my classes recently did a ski/snowboard project in which students skied down a run that was one mile long. We recorded the times for each run by each student. One of the times was 2 minutes and 22 seconds. We wanted to find the equation of the line. (We actually did graphs in Excel for each run by each student to find the equation of the line.) In this case, we will do it by hand. We want to first find the slope of the line. This slope will represent the average speed for the skier/rider. The units of the slope will be in mph. We already know the distance in miles is one. We have the time, which we will convert into hours. First we convert seconds to minutes. There are 60 seconds in a minute, we get $\frac{22}{60} = .366 \ldots$ So the total time in minutes is 2.366 minutes. We now convert to hours. There are 60 minutes in an hour, so we get: $\frac{2.366}{60} = .0394$ hours. Now we find the slope: $\frac{1}{.0394} = 25.38$ mph. We have two points (0,0), and (.0394, 1). How did we get these values? The first value, the x-value is the time; and the second value, the y-value represents distance. The (0,0) comes from our starting time and distance. When we began, we had traveled 0 miles at 0 hours. The equation of the line is $y = 25.38x$.

Exercise Set 4:

Complete the equation of the line using the above information for the following data: (Note we have already done the first problem. You can recopy it to self-review before completing the rest.

1) Hunter run 1: 2 min. 22 seconds
2) Hunter run 2: 2 min, 21 seconds
3) Josh run 1: 2 min, 45 seconds

4) Josh run 2: 2 min, 19 seconds
5) Michael run 1: 2 min, 47 seconds
6) Michael run 2: 2 min, 27 seconds
7) Aaron run 1: 2 min., 49 seconds
8) Aaron run 2: 2 min, 42 seconds

Example 7: Let us construct a financial example:

Let's say you deposit the same amount of money into your bank account each month. After four months of saving money, you have \$1000 in your bank account. After one year of saving money, you have \$5000. Find an equation that models how much money y, you will have after x months. We have 2 points (4,1000), (12,5000). $m = \frac{5000-1000}{12-4} = \frac{4000}{8} = 500 \rightarrow \quad y - 1000 = 500(x-4) \rightarrow \quad y = 500x + 3000.$

Exercise Set 5:

1) Lucy deposits the same amount of money into her bank account each month. After 6 months, she has \$20,000. After 2 years, she has \$100,000. Find an equation that models how much money y, she will have after x months.

2) Susan has \$7000 in her bank account. She anticipates spending \$200/month for the next year. Construct a linear model for the total amount of money y she will have over x months.

3) Jonathan has a shoe company. Find a linear model for the Revenue he brings in per units sold, if the price is \$20/shoe.

4) Find a linear model for the profit in number 3) if the cost is $y = 5x + 1000.$ Profit is Revenue minus Cost.

3. Math Modeling/Real World Applications: Annuities with applications

Goal: a. Complete understanding of the formula, b. Ability to Apply formula to some basic applications.

Definition: An Annuity is an account scheduled payments may be made in and out of.

Ordinary Annuity/Fixed Annuity: Equal payments made at regularly scheduled times. The interest is compounded at the end of each interval using a fixed interest rate.

Ordinary Annuity/Fixed Annuity Formula: $A = \frac{p\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}.$

Let us go through each part of the formula:

1) A is the accumulated value. This is the amount left in the account after t years.
2) p represents the payments in dollars made each time.
3) r is the interest rate.
4) n is the number of times per year the payments are made.
5) t is the number of years.

Example: The Johnsons are investing $1,000 for 20 years at 3% compounded quarterly. How much will they have at the end of 20 years?

First, we identify all the values that go into our formula:
A is the final value we are determining.
p is $1000.
r is 3% or .03
n is 4 (quarterly).
t is 20.

Next, we substitute all known values into our formula:

$$A = \frac{p\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}} = \frac{1000\left[\left(1+\frac{.03}{4}\right)^{4(20)} - 1\right]}{\frac{.03}{4}} = \frac{818.04}{.0075} = 109{,}072.$$

Note that we will go over the order of operations in a Calculator is section 2.

Exercise Set 6:

For the following set of exercises, find the values for p, r, n, t:

1) The Williams are investing $2,500 for 10 years at 5% compounded annually. How much will they have at the end of 10 years?
2) The Weinsteins are investing $1,000 for 5 years at 2.5% compounded monthly. How much will they have at the end of 5 years?
3) Fred is getting a 30 year fixed mortgage. If his payment is $4000/mo., and his interest rate is 3%, how much will he end up paying after 30 years? Use the same formula. A will be the total he pays over 30 years. (Again, just pick out the p, r, n, t.

Exercise Set 7:

Analyze the following formulas, by finding p, r, n, t:

1) $\dfrac{3700\left[\left(1+\frac{.025}{12}\right)^{12(20)}-1\right]}{\frac{.025}{12}}$

2) $\dfrac{8000\left[\left(1+\frac{.03}{2}\right)^{2(10)}-1\right]}{\frac{.03}{2}}$

3) $\dfrac{1750\left[\left(1+\frac{.035}{4}\right)^{4(25)}-1\right]}{\frac{.035}{4}}$

Exercise Set 8:

Find the accumulated value A, for the problems in exercise set 6.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 3.1., Chapter 11.6.
- Mathematical Excursions...
- Chapter 3.1, Chapter 10.2,
- Chapter 10.3

d. Required Topic 1d: Apply linear models and regression to real-world application problems. Applied geometry.

MAT-175 Required Topic 1d is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Application of linear models
2. Math Modeling/Real World Applications: Cost estimation using applied geometry

1. Math Modeling/Real World Applications: Application of linear models

Goal: a. To learn about the basic linear regression formula.

This is an optional section. I feel this would be a good extra credit problem. There will be a required section for Linear Regression using technology in Section 2c.

In this section, we will introduce you to the linear regression formula. We have covered other applications of linear models in the previous section. We will also cover linear regression in the technology section.

Let us start with the definitions:

Linear Regression: is to determine the linear relationship between 2 variables. Recall, in the previous section we used $y = mx + b$, the slope-intercept form of a line. In this section, we will have a variety of points. They may not exactly fall on a line, but they will be close. We will use what we call a "best fit line". This is easier to do using technology. The best fit is also called the least squares line, or least squares method.

Equations: $m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$, $\quad b = \frac{\sum y - m(\sum x)}{n}$.

Note that we must find m (slope), before finding b (y-intercept).

This formula is very complicated. Let us start with the symbol: $\sum$. This symbol is the upper-case Greek letter sigma. It means sum (or to add).

Example 1: $\sum i$, where $i = 1,2,3, \ldots$ This means to add up each number: $1 + 2 + 3 + 4 + \cdots$

In our formula, x is each of the x-coordinates we have, and y is each of the y-coordinates we have.

n is the number of points we have.

Example 2: (1,2), (2,4), (3,5.7), (4, 7.7). Let us find: n, x, y:

$n = 4$. We have 4 points (ordered pairs). $x = 1,2,3,4$. $y = 2,4,5.7,7.7$.

Exercise Set 1:

Find n, x, y: for the following problems:

1) (1,1), (2,2), (3,2.9), (4,4.01), (5,5).
2) (0,1.01), (1,2), (2,3), (3,4), (4,5.02).
3) (1,3), (2,6), (3,8.9), (4,12.2), (5,14.99)

Note: Calculator practice will come in section 2c.

Example 3: Let us find $\sum xy$ for Example 2. $\sum xy = (1 \cdot 2 + 2 \cdot 4 + 3 \ \cdot 5.7 + 4 \cdot 7.7) = 57.9.$

Let us find $\sum x$ for Example 2: $\sum x = 1 + 2 + 3 + 4 = 10.$

Let us find $\sum y = 2 + 4 + 5.7 + 7.7 = 99.4$

Let us find $\sum x^2 = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30.$

Let us find $(\sum x)^2 = (1 + 2 + 3 + 4)^2 = 10^2 = 100.$

Exercise Set 2: (Optional).

Find $\sum x, \sum y, \sum x^2, (\sum x)^2$ for all the problems in Exercise Set 1.

2. Math Modeling/Real World Applications: Cost estimation using applied geometry

We have covered this section in the previous section 1.c.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 13.8
- Mathematical Excursions...
- Chapter 13.5, Chapter 7.3

Required Topic 2a: Appropriate use of technology and internet resources: Introduction of selected technology such as scientific calculators, graphing calculators, Desmos, Wolfram Alpha, and Photomath.

MAT-175 Required Topic 2a is intended to support the following MAT-120 topics:

1. Formal Logic: Venn Diagrams
2. Math Modeling/Real World Applications: Graph analysis in the context of an application

1. Formal Logic: Venn Diagrams:

Goal: To demonstrate proficiency with a basic or scientific calculator. Intro to other technologies.

For applications using Venn Diagrams, a basic or scientific calculator will suffice. We will explore other technologies for later use.

Tutorials:

Ti-84: https://www.varsitytutors.com/hotmath/graphing_calculators/ti84_movie_index.html

Ti-89: https://www.youtube.com/watch?v=CVQ2qHX58Ak

Desmos.com Graphing Calculator Tutorial: https://learn.desmos.com/graphing

Wolframalpha.com Tutorial: https://www.wolframalpha.com/tour/

Photomath App Tutorial: https://www.photomath.net/en/

Scientific: https://www.youtube.com/watch?v=z0XkRIG4Snw

Scientific: https://www.wikihow.com/Operate-a-Scientific-Calculator

2. Math Modeling/Real World Applications: Graph analysis in the context of an application

Goal: To demonstrate proficiency in basic graphing using different technologies:

Tutorials:

Ti-84: https://youtu.be/q1OEXc_Gio4

Ti-89: https://youtu.be/H1GL6Vu9bZ4

Desmos: https://youtu.be/MYJ30F3c-jY

Excel: https://blog.hubspot.com/marketing/how-to-build-excel-graph

Required Topic 2b: Appropriate use of technology and internet resources: Graph linear and exponential functions.

MAT-175 Required Topic 2b is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Graph analysis in the context of an application
2. Math Modeling/Real World Applications: Construction of linear models

These were covered in the previous section.

Required Topic 2c: Appropriate use of technology and internet resources: Linear regression.

MAT-175 Required Topic 2c is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Construction of linear models

Ti-84: https://youtu.be/7v1-2kiGAEY

Ti-89: https://youtu.be/d-mx6-T8fO4

Desmos: https://youtu.be/zcZaI-xfiFE

Excel: This is an example I did in another class: (Project on next page.)

CLASS PROJECT
SKI/SNOWBOARD GRAPH INSTRUCTIONS

You will make a separate graph for each skier/snowboarder in Excel.

1) Convert each time from seconds to hour.
2) Plot distance on the y-axis and time on the x-axis.
   a) One point will be (0,0), the other will be (skier time, skier distance).
      i) Column A will be time on top, distance on the bottom and will be (0,0)
      ii) Column B will be skier time on top, distance on the bottom.
      iii) Highlight what you just typed.
      iv) At the top of the toolbar click on insert.
      v) Click on scatter.
      vi) Select scatter with lines.
      vii) Your linear graph will appear.
   b) Right click on the line.
   c) Click on add Trendline
   d) Trend/regression type should be linear.
   e) The second box from the bottom will say "Display equation on chart". Click that on.
   f) The equation of the line will now appear on your graph. **NOTE: YOU MUST HAVE THE EQUATION OF THE LINE ON ALL YOUR GRAPHS IN ORDER TO RECEIVE ALL THE POINTS.**
      The slope of your graph is the speed of the rider.
   g) At the top of Excel, click on Chart Tools.
   h) Click on Design
   i) Click on Chart Element.
   j) Click on Axes Titles.
      - Label your horizontal axis time, and your vertical axis distance.
   k) Now Click on Chart Title. Title your graph the name of the skier/snowboarder.
   l) Repeat process for the next skier/snowboarder.

Required Topic 2d: Appropriate use of technology and internet resources: Use of technology to evaluate models and formulas used in MAT-120 (e.g. graphing and scientific calculator).

MAT-175 Required Topic 2d is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Graph analysis in the context of an application

Goal: To be proficient in the use of technology for formulas used in MAT-120.

For graphical analysis, I feel we have covered this in the previous sections.

Here, I want to include a step-by-step guide to using a scientific calculator (this includes most phones) to evaluate common formulas used in MAT 120.

Let us start with the simple interest formula: $i = prt$. i is the interest accumulated, p is the principal, r is the interest rate, and t is the time in **years.** To calculate the interest, you simply multiply the 3 values together into the calculator in any order.

Let us now evaluate the compound interest formula. This is a formula that typically gives MAT 120 students difficulty if only a scientific calculator is available. In a graphing calculator, it is relatively simple.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A is the accumulated value, meaning the principal plus the interest you have accumulated after t years.

P is the principal. This is the money you originally invest.

r is the interest rate.

n is the number of times per year the interest is compounded.

t is time in years.

Example: Joyce wants to invest \$2000 at a rate of 3%, compounded monthly for 10 years. How much will she have in 10 years?

Let us substitute everything into our formula: $A = 2000\left(1 + \frac{.03}{12}\right)^{12(10)}$.

Let us now put this into our scientific (or phone) calculator. We must follow the order of operations:

1) Divide: $\frac{.03}{12} = .0025$
2) Next we add the 1: 1+.0025=1.0025
3) Next we multiply 12 times 10 = 120
4) We raise the 1.0025 to the 120th power: It is usually the x^y key on your calculator. We get 1.349…
5) Now we multiply that by $2000 to get: $2,698.71

Exercise Set 1:

Put the following problems into your calculator using the above steps to calculate:

1) $A = 2500\left(1 + \frac{.02}{12}\right)^{12(3)}$

2) $A = 1000\left(1 + \frac{.035}{4}\right)^{4(1)}$

3) $A = 3700\left(1 + \frac{.025}{2}\right)^{2(10)}$

4) $A = 2000\left(1 + \frac{.05}{12}\right)^{12(20)}$

Required Topic 2e: Appropriate use of technology and internet resources: Use of technology to perform statistics (e.g. scientific and graphing calculators).

MAT-175 Required Topic 2e is intended to support the following MAT-120 topics:

1. Probability and Statistics: Measures of Central Tendency
2. Probability and Statistics: Normal distribution

Ti-84: https://youtu.be/ShwFWQ0ikZw

Ti-89: https://youtu.be/qJU7TjUorf0

Desmos: https://youtu.be/SQT6RuPTxGs

Scientific (phone):

I want to include a detailed step-by-step instruction for calculating the standard deviation on a scientific calculator as many MAT 120 students struggle with this.

The formula is: $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$.

Let us go through the formula: s is the standard deviation (we will go through this in more detail in another section). x is each number in your set of data. $\bar{x}$ is the mean you must calculate. $\sum$ means sum (or to add). n is the number of entries in your set of data.

Example: Find the mean and standard deviation for the following set of data: 1,2,2,3,4,6:

First we find the mean: $\frac{1+2+2+3+4+6}{6} = 3.$

Next we can substitute everything into our formula. $s = \sqrt{\frac{(1-3)^2+(2-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(6-3)^2}{6.-1}}$.
Next, we will simplify the numerator under the radical. We can do this in our calculator or by inspection: $s = \sqrt{\frac{(-2)^2+(-1)^2+(-1)^2+0^2+1^2+3^2}{5}}$. We continue to simplify: $s = \sqrt{\frac{4+1+1+1+9}{5}} = \sqrt{\frac{16}{5}} =$ Now we put this into our scientific calculator: We divide $\frac{16}{5}$ to get: 3.2. We then hit the square root button to get: ≈ 1.789.

Required Topic 3: Study skills, and test taking strategies.

This is a list of topics. There will be separate documents covering all the details for these topics.

Apply satisfactory study skills

a. Incorporate Growth Mindset strategies in learning

b. Engage in appropriate math learning and testing strategies

c. Develop confidence in learning and doing mathematics

d. Identify academic support

e. Review as needed

Required Topic 4a: Understanding of mathematical notation: Review of Order of operations relating to sets and statements.

MAT-175 Required Topic 4a is intended to support the following MAT-120 topics:

1. Formal Logic: Subset, Union, intersection and complement

Goal: Demonstration of mastery of order of operations for sets and statements.

Definitions:

Complement: The Complement of a set A is called A prime, and is denoted by the symbol A'. This is the set of all elements in the Universal Set U, but **NOT** in the set A.

Example 1: Let $U = \{a, b, c, d, e), A = \{a, c, e\}, A' = \{b, d\}$.

Intersection: The intersection between two sets, we will call A and B, is denoted by the symbol $\cap$. So $A \cap B$ reads A intersect B. This is the set containing both A **and** B. This is the overlapping region, or what is common to both sets.

Example 2: Let $U = \{a, b, c, d, e), A = \{a, c, e\}, B = \{c, d, e\},\ A \cap B = \{c, e\}$.

Union: The union between two sets, we will call A and B, is denoted by the symbol $\cup$. So $A \cup B$ reads A union B. This is the set containing both A **or** B, or both.

Example 3: Let us use the same sets from Example 2. Then $A \cup B = \{a, c, d, e\}$.

Order of Operations:

We start with parentheses, just as we do with real numbers. **Example:** Using the sets from Example 2 again, let us find $(A \cup B) \cap A'$. We must first find $A \cup B$. In Example 3, we found it to be $A \cup B = \{a, c, d, e\}$. Next, we find A'. In Example 1, we found it to be $A' = \{b, d\}$. Now we put it together: $\{a, c, d, e\} \cap \{b, d\} = \{\}$. $\{\} = \emptyset$ denotes the empty set. This is the set that contains no elements. We note that A' has nothing in common (no overlap) with $A \cup B$.

Example 4:

Let us list the order of operations for the following sets: (Note: we will not determine the sets studied).

1) $(A \cup B) \cup B'$:
   a. $A \cup B$.
   b. B'
   c. $(A \cup B) \cup B'$

2) $(A \cap C) \cup (B \cap C)$:
   a. $A \cap C$
   b. $B \cap C$
   c. $(A \cap C) \cup (B \cap C)$:

   Note: For both 1) and 2), a. and b. could be reversed (meaning we could do b. then a.).

Exercise Set 1:

For the following set of exercises, list the order of operations as we did in Example 4:

1) $(A \cup B) \cup C'$:
2) $(A \cap B) \cup (C \cup A')$:
3) $(A' \cup C') \cap (B \cap A)$:
4) $B \cup (C \cap A')$:
5) $(B \cup A) \cap (C \cup D) \cup (A' \cap B')$:

Logic Statements:

These will follow from what we learned from set theory above. The symbols are different, but the order of operations still follow the same set of rules.

We will construct statements using the letters: p, q, r. Each letter represents a simple statement. We will join them together to make compound statements (two or more simple statements using the following connectives: **and, or, if…then, if and only if.**

We denote these as follows: **and:** $\wedge$, **or:** $\vee$, **if…then:** $\rightarrow$, **if and only if:** $\leftrightarrow$.

Negation (or NOT statements) are denoted by the symbol ~.

Example 5:

List the order of operations:

1) $p \vee (q \wedge r)$:
   $a.$ $q \wedge r$
   b. $p \vee (q \wedge r)$

2) $(p \wedge q) \vee (p \vee r)$:
   $a.$ $p \wedge q$
   b. $p \vee r$
   c. $(p \wedge q) \vee (p \vee r)$

Note: a. and b. could be reversed.

3) $\sim p \wedge (q \vee r)$:
   a. $\sim p$
   b. $q \vee r$
   c. $\sim p \wedge (q \vee r)$

Note: a. and b. could be reversed.

4) $\sim (p \vee r) \wedge q$
   a. $p \vee r$
   b. $\sim (p \vee r)$
   c. $\sim (p \vee r) \wedge q$

Note: We had to find $p \vee r$ first, before negating the statement, since $p \vee r$ was in parentheses. Then we negate that entire statement before moving on.

Exercise Set 2:

For the following set of exercises, list the order of operations as we did in Example 5:

1) $p \wedge (q \vee r)$
2) $(p \vee q) \wedge (p \vee r)$
3) $(p \wedge \sim r) \vee q$
4) $(\sim p \vee \sim r) \wedge (q \wedge r)$
5) $\sim (p \wedge r) \vee \sim (q \vee r)$

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8^{th} ed.
    - Chapter 2.3, 3.1
- Mathematical Excursions...
- Chapter 2.3, Chapter 3.1
- Chapter 3.2

Required Topic 4b: Understanding of mathematical notation: Review of mathematical notation as it applies to MAT120.

MAT-175 Required Topic 4b is intended to support the following MAT-120 topics:

1. Formal Logic: Subset, Union, Intersection and Complement
2. Formal Logic: Introduction to Conditional & Biconditional
3. Formal Logic: Introduction to Converse, Inverse, & Contrapositive
4. Math Modeling/Real World Applications: Direct and inverse variation
5. Probability and Statistics: Measures of Variation
6. Probability and Statistics: Normal distribution

Goal: To have a substantial level of comfort and familiarity with all the common mathematical notation used in MAT120.

1. **Formal Logic: Subset, Union, intersection and complement:**

Definitions:

Subset: A set A is a subset of a set B if and only if all the elements in A are also in B. We denote the subset with the symbol: $\subseteq$. **Example:** Let $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$. $A \subseteq B$, because all the elements in A are also in B. **Example:** Let $A = \{1,2,3\}$, $B = \{3,2,1\}$. $A \subseteq B$, because all the elements in A are also in B. Note also that $B \subseteq A$, because all the elements in B are also in A.

A set A is equal to a set B if they contain all the same elements. **Note:** $A = B$ in the previous example.

Proper Subset: A set A is a proper subset of a set B if and only if all the elements in A are also in B, and set $A \neq$ set B. **Note:** This indicates that the set A is smaller than the set B, that it always contains fewer elements.

Note: Notice the similarity of the symbols $\subset$, and $\subseteq$, to the symbols $<$ and $\leq$, from our familiarity with real numbers. Strictly less than, and less than or equal to with the equal part on the bottom of the symbol. The symbols $\subset$, and $\subseteq$ follow a similarity for sets.

Note: The empty set denoted by $\{\}$ or $\emptyset$ (which we covered briefly in the previous section, is a subset of all sets.

Example 1: Let $A = \{a, b, c, d, e\}, B = \{c, d, e\}$. Decide if $A \subseteq B,\ B \subseteq A,\ A \subset B,\ B \subset A,\ A = B$.

We will list all that are true. We observe that $B \subseteq A$ and that $B \subset A$. We notice that none of the others are true.

Exercise Set 1:

For this set of exercises decide whether: $A \subseteq B,\ B \subseteq A,\ A \subset B,\ B \subset A,\ A = B$. List all that are true.

1) Let $A = \{1,2,3,4,5\},\ B = \{2,3,1,5,4\}$
2) Let $A = \{4,5,1\},\ B = \{1,2,3\}$
3) Let $A = \{1,2,3,4,5\},\ B = \{2,5,3\}$
4) Let $A = \{1\},\ B = \{1,4,5\}$
5) Let $A = \{\},\ B = \{1,3\}$.

Union, intersection and complement:

We covered these under Order of Operations in the previous section. Let us review the definitions:

Complement: The Complement of a set A is called A prime, and is denoted by the symbol A'. This is the set of all elements in the Universal Set U, but **NOT** in the set A.

Example 2: Let $U = \{a, b, c, d, e), A = \{a, c, e\},\ then\ \ A' = \{b, d\}$.

Intersection: The intersection between two sets, we will call A and B, is denoted by the symbol $\cap$. So $A \cap B$ reads A intersect B. This is the set containing both A **and** B. This is the overlapping region, or what is common to both sets.

Example 3: Let $U = \{a, b, c, d, e), A = \{a, c, e\}, B = \{c, d, e\},\ then\ \ A \cap B = \{c, e\}$.

Union: The union between two sets, we will call A and B, is denoted by the symbol $\cup$. So $A \cup B$ reads A union B. This is the set containing both A **or** B, or both.

Example 4: Let us use the same sets from Example 2. Then $A \cup B = \{a, c, d, e\}$.

In the previous section, we went over the order of operations for these operations.

In this section, we will perform some very basic examples. These will be covered in more detail in MAT120.

Exercise Set 2:

You can use Examples 2-4 above as a reference for this exercise set.

In this exercise set, use $U = \{1,2,3,4,5,6,7,8,9\},\ A = \{1,3,5,7,9\},\ B = \{2,4,6,8\},\ C = \{1,4,9\}$

1) Find $A \cup B$
2) Find $A \cap B$
3) Find A'
4) Find B'
5) Find C'
6) Find $A \cup C$

7) Find $B \cap C$
8) Find $A \cap B \cap C$
9) Find $A \cup B \cup C$

2. Formal Logic: Introduction to Conditional & Biconditional

We briefly introduced Conditional and Biconditional in the previous section. We did not formally define it.

Conditional: if…then: $\rightarrow$. This means **if** the first statement is true, **then** the second statement is also true.

Biconditional: if and only if: $\leftrightarrow$ This implies: **if** the first statement is true, **then** the second statement is also true, **and** : **if** the second statement is true, **then** the first statement is also true.

In this section, we will learn to use the basic formulas to create very basic examples. More complex problems (including the use of commas) will be taught in MAT 120.

Example 1: Let p: It snows.

q: I will take you snowboarding.

Let's construct a conditional statement using symbolic language, and use the notation to represent it.

If it snows then I will take you snowboarding: $p \rightarrow q$. (Note: the absence of a comma is intentional).

Now, we will construct a bi-conditional statement out of it:

I will take you snowboarding if and only if it snows: $q \leftrightarrow p$. This one means: If I take you snowboarding then it will snow, and if it snows I will take you snowboarding.

Note: This makes more common sense in one direction. It does not matter. Learning logic may not follow from common sense. It merely has to follow the rules.

Example 2: Write a statement in symbolic language to denote the following:

If it is sunny I will go for a hike.

Let p: It is sunny.

q: I go for a hike.

Statement: $p \rightarrow q$.

Example 3: Write a statement in symbolic language to denote the following:

If it is not raining then I will go outside.

First: We recall from the previous section, that negation (NOT) is denoted by ~.

Let p: It is raining.

q: I will go outside.

Statement: $\sim p \rightarrow q$.

Exercise Set 3:

In this exercise set, define the variables, and put the sentence into symbolic form:

1) If you are nice, I will take you for ice cream.
2) I will go bike riding if and only if it rains.
3) Sue will go to the store if and only if she buys a car.
4) Brandy will not be happy if there is an earthquake.
5) Joey will not walk his dog if and only if it is hailing outside.

3. Formal Logic: Introduction to Converse, Inverse, & Contrapositive

Definitions:

Variations of the Conditional Statement:

Conditional: $p \rightarrow q$	If p then q
Converse of the conditional: $q \rightarrow p$	If q then p
Inverse of the conditional: $\sim p \rightarrow \sim q$	If not p then not q
Contrapositive of the conditional: $\sim q \rightarrow \sim p$	If not q then not p

Example 1:

In this example we will identify which of the above statement applies:

Let p: Joyce will walk her dog.

q: It is a beautiful day.

If it is not a beautiful day, Joyce will not walk her dog.

We observe that it is the Contrapositive: $\sim q \rightarrow \sim p$

Example 2:

In this example we will identify which of the above statement applies:

Let p: It is peaceful outside.

q: George is happy.

If it is not peaceful outside, George is not happy.

We observe this is the Inverse: $\sim p \rightarrow \sim q$

Exercise Set 4:

In this exercise set, determine whether the statement is a Converse, Inverse, or Contrapositive:

For each exercise: Let p: It is an exciting lunch.
q: John is having spaghetti.

1) If John is having spaghetti then it is an exciting lunch.
2) If John is not having spaghetti then it is not an exciting lunch.
3) If it is not an exciting lunch then John is not having spaghetti.

For the next exercise: Let p: Sally is buying a snowmobile.
q: Sally hopes it snows.

4) If Sally does not hope it snows then Sally is not buying a snowmobile.
5) If Sally is not buying a snowmobile then Sally does not hope it snows.
6) If Sally hopes it snows then Sally is buying a snowmobile.

4. Math Modeling/Real World Applications: Direct and inverse variation

We covered this section in Required Topic 1b: Go back and review that section.

5. Probability and Statistics: Measures of Variation:

Note: The reason we are not covering Measures of Central Tendency here, is because the formulas are simple, and we covered how to enter them into different forms of technology in Section 2e.

Definitions:

Range: Highest Value – Lowest Value. This is fairly straightforward.

Example 1: Our list of data: 16,15,17,19,25,12.

Range: Highest – Lowest: 25-12=13.

Standard Deviation: We covered this in Section 2e. Let us review:

$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$.

Let us go through the formula: s is the standard deviation (we will go through this in more detail in another section). x is each number in your set of data. $\bar{x}$ is the mean you must calculate. $\sum$ means sum (or to add). n is the number of entries in your set of data.

Example 2:

Find the mean and standard deviation for the following set of data: 1,2,2,3,4,6:

First we find the mean: $\frac{1+2+2+3+4+6}{6} = 3.$

Next we can substitute everything into our formula. $s = \sqrt{\frac{(1-3)^2+(2-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(6-3)^2}{6.-1}}$.
Next, we will simplify the numerator under the radical. We can do this in our calculator or by inspection: $s = \sqrt{\frac{(-2)^2+(-1)^2+(-1)^2+0^2+1^2+3^2}{5}}$. We continue to simplify: $s = \sqrt{\frac{4+1+1+1+9}{5}} = \sqrt{\frac{16}{5}} =$ Now we put this into our scientific calculator: We divide $\frac{16}{5}$ to get: 3.2. We then hit the square root button to get: $\approx 1.789.$

Exercise Set 5:
In this exercise set, complete the following sets for each set of data:

1) Find the mean $\bar{x}$.
2) Subtract each x-value from $\bar{x}$. (Note: the x-values are the numbers in your list).
3) Square all the new values found in step 2).
4) Add up all the numbers in step 3).
5) Count how many numbers you have.
6) Subtract 1 from the number in step 5).
7) Divide the number found in step 4), by the number you found in step 6).
8) Take the square root of what you found in step 7).

Here are the lists of data:
a) 1,2,3,4,5,6,7,8
b) 5,7,11,14,16,2
c) 10,11,12,17

7. Probability and Statistics: Normal distribution

Normal Distributions are graphs which will be covered in that section.

For this section on Formulas we will briefly cover **z-Scores.**

$z = \frac{value\ of\ the\ piece\ of\ data - mean}{standard\ deviation} = \frac{x-\mu}{\sigma}$.

Example: Let us return to the previous example yet again: Find the mean and standard deviation for the following set of data: 1,2,2,3,4,6:

First we find the mean: $\frac{1+2+2+3+4+6}{6} = 3.$

Next we can substitute everything into our formula. $s = \sqrt{\frac{(1-3)^2+(2-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(6-3)^2}{6.-1}}$. Next, we will simplify the numerator under the radical. We can do this in our calculator or by inspection: $s = \sqrt{\frac{(-2)^2+(-1)^2+(-1)^2+0^2+1^2+3^2}{5}}$. We continue to simplify: $s = \sqrt{\frac{4+1+1+1+9}{5}} = \sqrt{\frac{16}{5}} =$ Now we put this into our scientific calculator: We divide $\frac{16}{5}$ to get: 3.2. We then hit the square root button to get: $\approx 1.789.$

Next: Let us find the z-score for the value $x = 1$: We have $x = 1$, we have the mean: $\bar{x} = \mu = 3.$ We have the standard deviation $s = \sigma = 1.789.$ Therefore $z = \frac{1-3}{1.789} = -1.1179$ for $x = 1.$

Exercise Set 6:

Find the z-score for the first three numbers in Exercise set 5a:

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapters 2.2, 2.3, 3.1-3.4, 13.6-13.7
- Mathematical Excursions...
- Chapter 2.2, Chapter 2.3
- Chapter 3.3, Chapter 3.4
- Chapter 13.3, Chapter 13.4

Required Topic 5a: Techniques for graphing functions without the use of technology: Intro to different graphing techniques.

MAT-175 Required Topic 5a is intended to support the following MAT-120 topics:

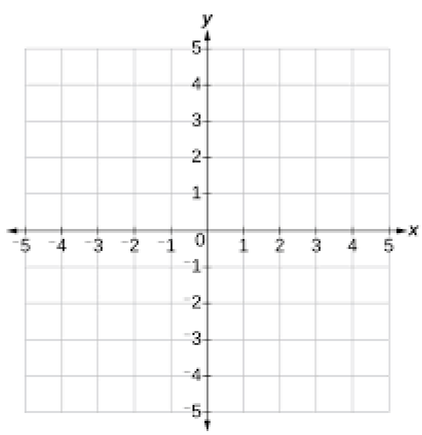
1. Math Modeling/Real World Applications: Graph analysis in the context of an application
2. Probability and Statistics: Normal distribution

1. Math Modeling/Real World Applications: Graph analysis in the context of an Application

Goal: Proficiency in basic graphing techniques without technology that may be applied to areas of interest in MAT 120.

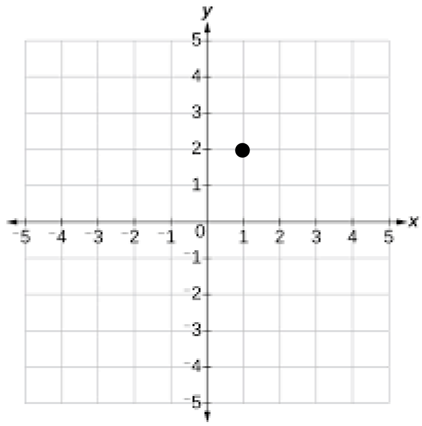
Method 1: Point Plotting:

Review of Ordered Pairs: Ordered pairs have an x-coordinate, and a y-coordinate: (a, b), where a is any real number and is the x-coordinate, and where b is any real number and is the y-coordinate. We can plot points on a Cartesian Coordinate System. This is also referred to as a Rectangular Coordinate system, where the x-axis is the horizontal axis where $y = 0$, and where the y-axis is the vertical axis where $x = 0$.



Let us plot the point (1,2).

We start at the origin (0,0), go right 1, and up 2:



What if we want to graph more than a point? Let's try some other equations. We will start by using the basic method of Point-Plotting:

We will make a "T-Chart"

x	y

How do we use such a chart? We pick random values for x, and then use our equation to find the y-value for each x-value.

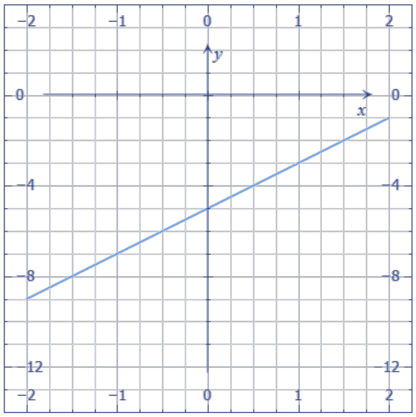
Example 1:

Let $y = 2x - 5$:

Let us find some values for x, and calculate each y, and fill out the following T-Chart:

x	y
-2	-9
-1	-7
0	-5
1	-3
2	-1
3	1
4	3
5	5

Now we plot the points and graph: (We plot each point as we did in the brief example before this one, and connect the dots.



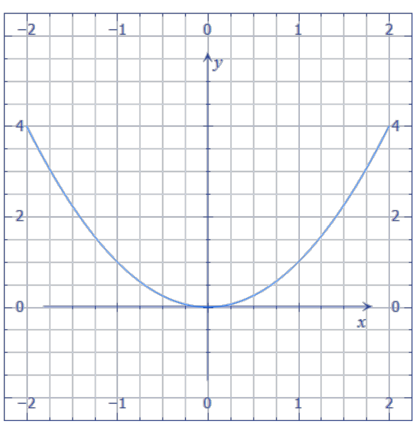
Example 2:

Let $y = x^2$:

Let us find some values for x, and calculate each y, and fill out the following T-Chart:

x	y
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16
5	25

Now we plot the points and graph:



Exercise Set 1:

Graph the following set of equations by filling out a T-Chart, plotting each point, and connecting all the points;

1) $y = 2x$:
2) $y = x - 5$:
3) $y = 3x + 1$:
4) $y = 2x^2$:
5) $y = x^2 - 5$:
6) $y = x^3$:
7) $y = x^2 - 2x + 1$:

Method 2: Graphing by finding the x and y-intercepts:

This method works best for linear equations. (**Note:** For graphing a line, we only need to find 2 points).

Let us review x and y-intercepts:

An x-interecept is where the graph crosses the x-axis. This occurs when $y = 0$.

An y-interecept is where the graph crosses the y-axis. This occurs when $x = 0$.

To find an x-interecept, we set $y = 0$, and solve for x.

To find an y-interecept, we set $x = 0$, and solve for y.

Example 1:

Find the x and y intercepts for the equation $2x + 4y = 8$.

x-interecept: We set $y = 0$. $2x + 4 \cdot 0 = 8 \rightarrow \quad 2x = 8 \rightarrow \quad x = 4$.

y-interecept, we set $x = 0$. $2 \cdot 0 + 4y = 8 \rightarrow \quad 4y = 8 \rightarrow \quad y = 2$.

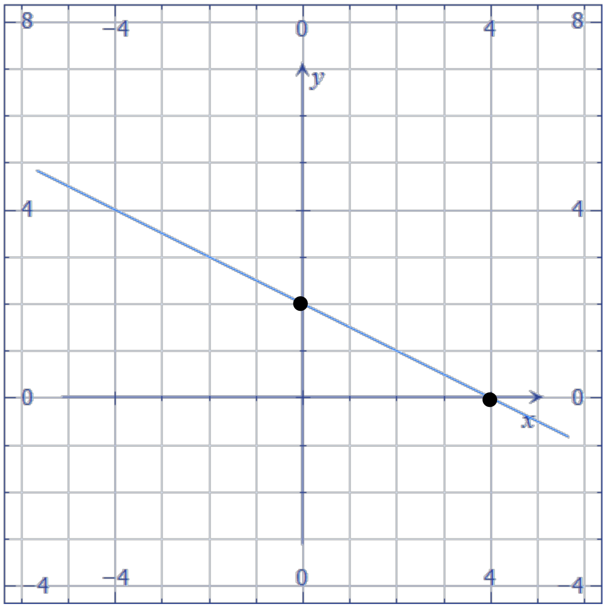
Example 2:

Find the x and y intercepts for the equation $x - 3y = 12$.

x-interecept: We set $y = 0$. $0 - 3y = 12 \rightarrow \quad -3y = 12 \rightarrow \quad y = -4$.

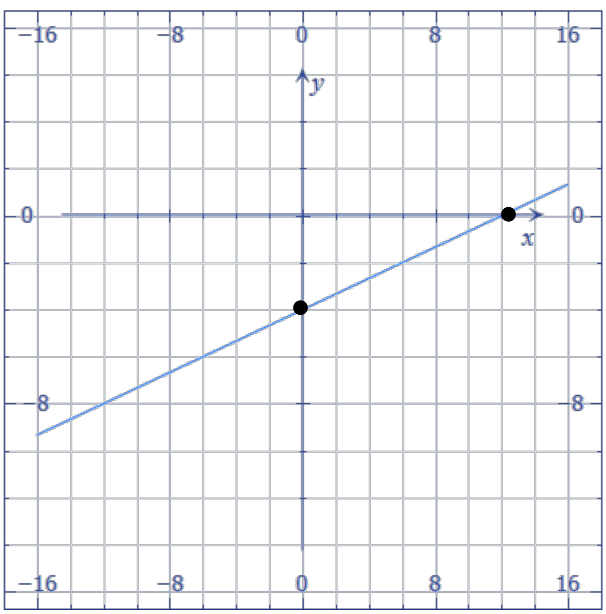
y-interecept, we set $x = 0$. $x - 3 \cdot 0 = 12 \rightarrow \quad x = 12$.

Example 3:
Graph the equation in Example 1 by plotting the x and y intercepts, and drawing a line through the points.



Example 4:

Graph the equation in Example 1 by plotting the x and y intercepts, and drawing a line through the points.



Example 5: Let $y = 2x$.

We quickly note that the x and y intercept are both 0. I.e., this line goes through the origin (0,0). That is only one point. We will have to find another point to graph this line. You can choose any x-value, and find the y-value as we did in the point-plotting method.

Exercise Set 2:

For the following set of exercises, find the x and y interecepts.

1) $y = 2x - 6$

2) $2x + 2y = 8$

3) $x - y = 9$

4) $\frac{1}{2}y - x = 2$

5) $x = y - 7$

Exercise Set 3:

For the following set of exercises, graph the equations in Exercise Set 2 by finding the x and y intercepts as we did in Examples 3,4.

2. Probability and Statistics: Normal distribution

Normal (or Gaussian) distribution has a graph that is a histogram, where the mean, median and mode all have the same value.

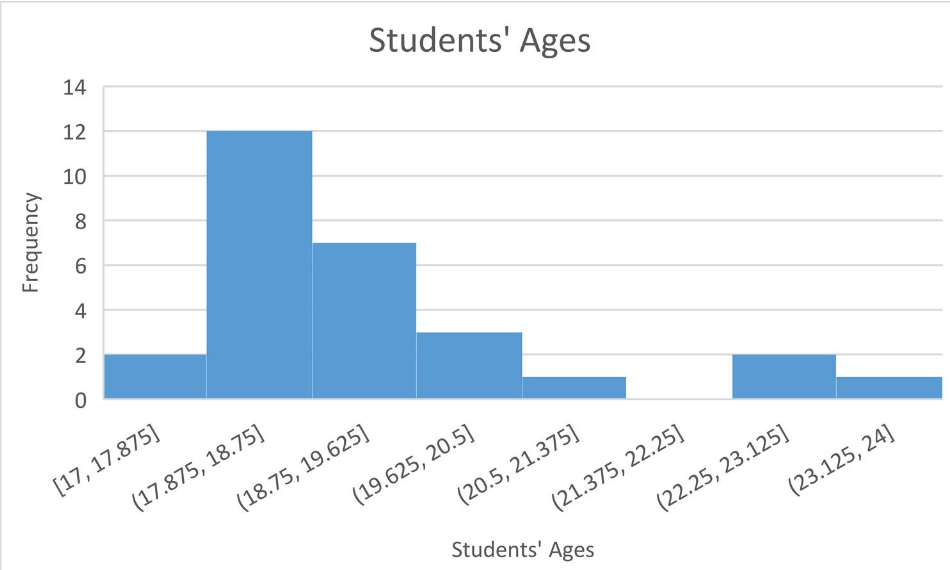
Let us review how to graph a **Histogram:**

A Histogram is a statistical graph that illustrates a frequency distribution. This is sometimes referred to as a bar graph. Along the x-axis (the horizontal axis), we plot the observed values (or group of values: classes) given. Along the y-axis we plot the frequency of the value(s), i.e. the number of time said value(s) occur.

Example: Observe the following frequency distribution:

Ages of Students	# of Students of that age
17	2
18	12
19	7
20	3
21	1
22	0
23	2
24	1

Let us make a Histogram for the following set of data;



Exercise Set 3:

Since this is a graphing without technology section, graph histograms by hand for the following set of data:

1)

Ages of Preschoolers	# of Students of that age
3	13
4	5
5	10

2)

Student Test Scores	# of Test Scores in that Range
95-100	3
89-94	10
83-88	7
77-82	12
71-76	11
65-70	1
59-64	0
53-58	2
Below 53	2

3)

Temperature	# of Days in June having that Temp
70-75	3
76-81	8
82-87	11
88-93	7
94-99	1

4)

# Burritos Sold	# Days in August Sold
42	3
53	4
54	5
57	2
58	6
59	3
60	6
63	2

Again, the normal distribution is a specific type of Histogram that you will learn more about in MAT 120.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 13.4
- Mathematical Excursions...
- Chapter 10.1, Chapter 10.2,
- Chapter 13.1, Chapter 13.4

Required Topic 5b: Techniques for graphing functions without the use of technology: Graphing of linear and exponential functions.

MAT-175 Required Topic 5b is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Graph analysis in the context of an application.
2. Math Modeling/Real World Applications: Appropriate use of linear and exponential models.
3. Math Modeling/Real World Applications: Construction of linear models.

Goal: For these 3 topics, we will demonstrate proficiency in graphing linear and exponential functions. The construction and application of these models will be taught in MAT 120.

Linear Functions: In the previous section, we have learned 2 methods for graphing Linear Functions: 1) Point-plotting, and 2) Finding intercepts.

In this section, we will learn one more technique: Using Slope-Intercept Form.

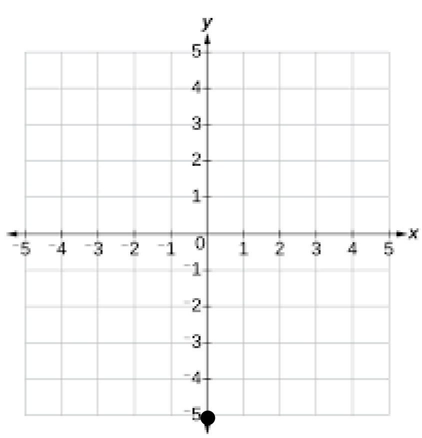
Definition: Recall the equation of a line in slope-intercept form is: $y = mx + b$, where m is the slope of the line, and b is the y-intercept.

How do we graph a line in this form: We start with the y-intercept. We plot this point. We then use the slope to find another point. (Recall, we need only 2 points to graph a line).

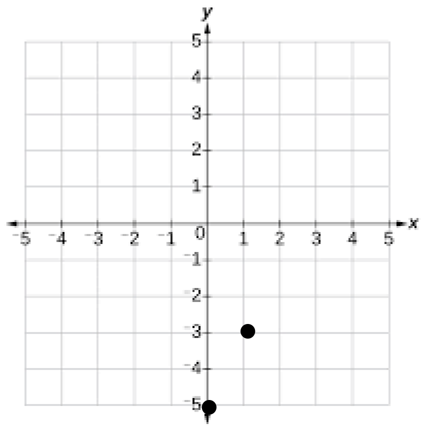
Recall the slope is $\frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. Recall y goes down to up, x goes left to right. If the slope is negative we go down by y-units, and over (right) by x-units. If the slope is positive, we go up by y-units, and over by x-units. **Note:** (If m is negative, we could also go up by y-units, and left by x-units. It is the same thing.) Let us stick with the aforementioned so as not to get confused.

Example 1: Let $y = 2x - 5$. Let us graph this:

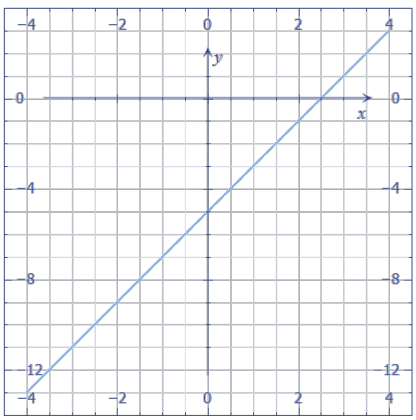
First: We plot the y-intercept: (0,-5).



Next, we plot another point. The slope is 2, so we go up 2 and over 1. (**Note:** Since the denominator is 1, that is the value of x we move to the right).



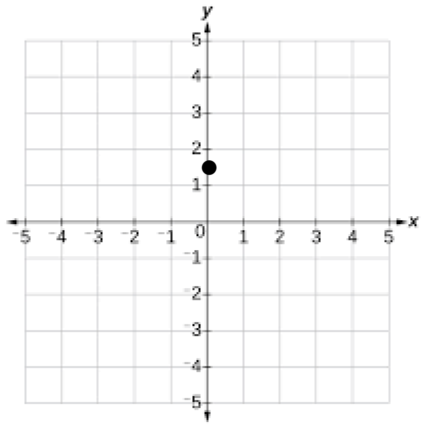
Next, we connect the dots:



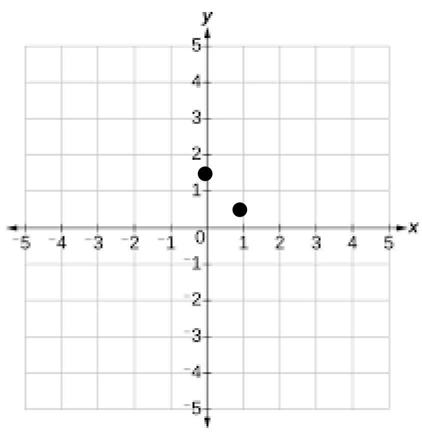
Example 2: Let $2y + 2x = 3$. Let us graph this:

In this case, we must first put our equation into slope-intercept form: We must solve for y. $y = -x + \frac{3}{2}$.

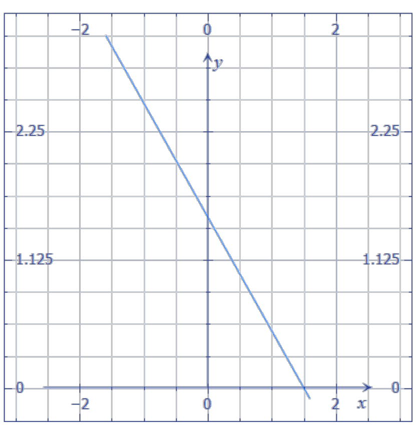
We plot the y-intercept: $\left(0, \frac{3}{2}\right)$.



Now we plot another point using the slope: since $m = -1,$ we go down 1, and over to the right 1.



Next: We connect the dots:



Exercise Set 3:

For the following set of exercises, graph the linear equation by using slope-intercept form: Note: You may have to put it into this form first:

1) $y = x - 2$
2) $y = -3x + 1$
3) $y = 2x$
4) $y = \frac{1}{3}x - 3$
5) $2x - y = 5$
6) $x + 2y = 1$
7) $2x - 3y = 0$
8) $y = 3$

Exponential Functions:

Previously, we discussed what a function is. We recall that for an equation to be a function, each x-value, we have exactly one y-value.

A test to determine if the a graph of a relation is a function is called **The Vertical Line Test:** To use this test for a graph of a relation, we draw a vertical line anywhere on the graph. If it intersects the graph in only one place, it is a function. The exponential equation: $y = a^x$, where $a > 0, a \neq 1$, satisfies this test. Therefore the exponential equation: $y = a^x$ is a function. We could use the notation, $f(x) = a^x$, but we will save that notation for your MAT 120 class.

Let us briefly discuss why $a > 0, a \neq 1$. If $a = 0$, or if $a = 1$, we would not have an exponential function, but a linear one, as 0 times itself over and over is 0, and likewise for 1. If $a < 0$, we would get a lot of imaginary numbers, which is not what we want.

Let us discuss **2 methods** for graphing exponential functions (or equations). First, we will go back to our point plotting method:

Example 3 : Let $y = 2^x$:

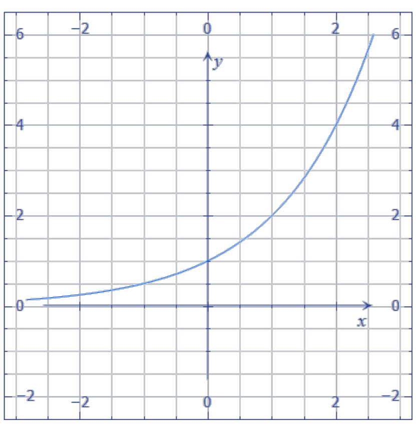
Let us make another T-chart:

We will pick random values for x, and calculate each y-value.

Recall that when x is negative, by rules of exponents, we get the reciprocal of our value, i.e., recall: $a^{-n} = \frac{1}{a^n}$. For example: $2^{-2} = \frac{1}{4}$.

x	y
-4	$\frac{1}{16}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	0
1	2
2	4
3	9
4	16

Next, we will plot the points and graph:

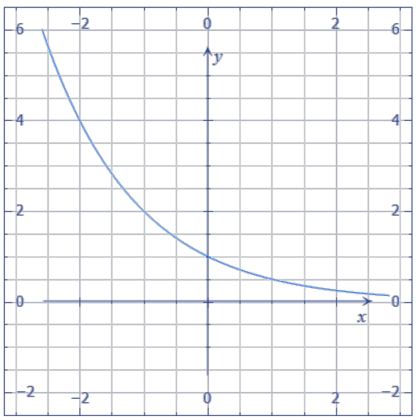


Example 4: Let $y = \left(\frac{1}{2}\right)^x$. Note that this is the same as $y = 2^{-x}$.

We will do another T-chart:

x	y
4	$\frac{1}{16}$
2	$\frac{1}{4}$
1	$\frac{1}{2}$
0	0
-1	2
-2	4
-3	9
-4	16

Now we graph:



Exercise Set 4:

For this set of exercises, graph the following exponential equations by using the point-plotting method:

1) $y = 3^x$
2) $y = \left(\frac{1}{3}\right)^x$
3) $y = 5^{-x}$
4) $y = 10^x$

Method 2:

In this method, we will use the fact that an exponential function always goes through the point (0,1) (unless it has a shift, which we will not cover in the graphing portion of this course), will always be constantly increasing or, constantly decreasing, and will have a horizontal asymptote at the x-axis, or $y = 0$. The asymptote, in this case, is a line in which the function will get closer and closer, but will not cross or touch. In this case, it will only approach the asymptote in one direction, depending whether the function is increasing or decreasing.

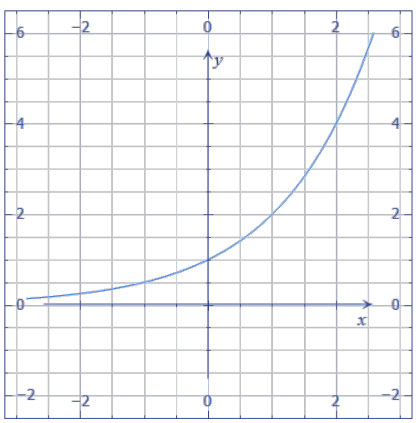
From Method 1, we observed that when $a > 1$, the function was increasing everywhere. It was getting closer and closer to the asymptote $y = 0$ on the left side of the graph. The further left we went, the closer it got. On the right side, it got larger and larger the farther you traveled. If you think about this, it makes sense. If you have a number >1, and you keep multiplying it by itself over and over, it will grow rapidly. When x is negative, it gives the reciprocal, so that when multiplying it by itself over and over, it decreases rapidly until it gets very close to zero. We observe it can never by 0 or negative. When $0 < a < 1$, we observe the opposite is true. We also recall that $a > 0, a \neq 1$.

We also observe that they all go through the point (0,1). Recall that any number to the 0 power is always 1. Why? We can write any number $\neq 0$ as something divided by itself. Let $1 = \frac{a^n}{a^n}$. By rules of exponents, we subtract the $n's$. $\frac{a^n}{a^n}$ becomes $a^{n-n} = a^0$. Which we had originally defined to be 1.

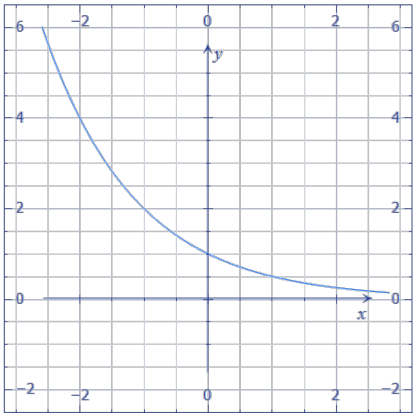
To summarize:

(Without any kind of shifts or transformations→ not covered here).
1) Exponential equations all have the point (0,1).
2) Exponential equations have a horizontal asymptote at the x-axis, $y = 0$.
3) Exponential equations are increasing everywhere when $a > 1$, and decreasing everywhere when $0 < a < 1$.

We use this to graph the equations we had in method 1:

$y = 2^x$: 

And $y = \left(\frac{1}{2}\right)^x = 2^{-x}$:



Exercise Set 5:

For this set of exercises, graph the exponential equations in Exercise Set 4 by using Method 2:
Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8^{th} ed.
  - Chapter 6.10
- Mathematical Excursions…
- Chapter 10.2, Chapter 10.5

Required Topic 6a: Analyze, differentiate and evaluate commonly used formulas.: Basic review of exponential models with a complete breakdown of each component.

MAT-175 Required Topic 6a is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Appropriate use of linear and exponential models.
2. Math Modeling/Real World Applications: Application of basic exponential models

Goal: For these 2 topics, we will demonstrate proficiency in understanding exponential models, and their formulas. The application will be further explored in MAT 120. We will do a few basic applications here.

Khan Academy Support: https://www.khanacademy.org/math/algebra-home/alg-exp-and-log/alg-intro-to-rate-of-exponential-growth-and-decay/v/word-problem-solving-exponential-growth-and-decay

Exponential Functions: We just did a basic explanation of exponential functions in the previous section.

To recap:

The exponential equation: $y = a^x$, where $a > 0, a \neq 1$, satisfies the vertical line test (covered in the previous section). Therefore the exponential equation: $y = a^x$ is a function. We could use the notation, $f(x) = a^x$, but we will save that notation for your MAT 120 class.

Let us briefly discuss why $a > 0, a \neq 1$. If $a = 0$, or if $a = 1$, we would not have an exponential function, but a linear one, as 0 times itself over and over is 0, and likewise for 1. If $a < 0$, we would get a lot of imaginary numbers, which is not what we want.

An exponential function always goes through the point (0,1), will always be constantly increasing or constantly decreasing, and will have a horizontal asymptote at the x-axis, or $y = 0$. The asymptote, in this case, is a line in which the function will get closer and closer, but will not cross or touch. In this case, it will only approach the asymptote in one direction, depending whether the function is increasing or decreasing.

We observed that when $a > 1$, the function was increasing everywhere. It was getting closer and closer to the asymptote $y = 0$ on the left side of the graph. The further left we went, the closer it got. On the right side, it got larger and larger the farther you traveled. If you think about this, it makes sense. If you have a number >1, and you keep multiplying it by itself over and over, it will grow rapidly. When x is negative, it gives the reciprocal, so that when multiplying it by itself over and over, it decreases rapidly until it gets very close to zero. We observe it can never by 0 or negative. When $0 < a < 1$, we observe the opposite is true. We also recall that $a > 0, a \neq 1$.

Basic Formula:

Let us begin by breaking down the basic exponential formula: $y = a^x$. We are already familiar with y as the output, or **dependent**, variable. a is the base. The base is the number you will multiply over and over x times. x is the input, or **independent**, variable. We choose values for x, and find y based on our choice for x. We note that a, the base, is a constant. A constant is a number, a value that does not change. a could be 2 for example. Or it could be $\frac{1}{3}$. a can be any real number except a must be >0, and recall that $a \neq 1$. It can be any number other than the restrictions listed. x can be any real number. We call these values the **Domain.** These are all the allowable input values. y will always be >0, or $(0, \infty)$. We call these values the **Range.** These are all the possible output values we could obtain.

Review: Let us quickly review some basic rules of exponents:

1) $a^m \cdot a^n = a^{m+n}$
2) $\frac{a^m}{a^n} = a^{m-n}$
3) $(a^m)^n = a^{m \cdot n}$
4) $a^{-n} = \frac{1}{a^n}$
5) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
6) $(ab)^n = a^n \cdot b^n$
7) $\left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

Example 1:

Evaluate: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$

Evaluate: $\left(\frac{1}{3}\right)^5 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{243}$

Evaluate: $4^2 \cdot 4^3 = 4^5 = 1024$ (using exponent rule 1)).

Evaluate: $\frac{3^2}{3^5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ (using exponent rules 2) and 4)).

Evaluate: $(2^2)^4 = 2^8 = 256$ (using exponent rule 3)).

Evaluate: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$ (using exponent rules 5) and 7)).

Exercise Set 1:

Evaluate the following:

1) 2^6
2) 3^3

3) $\left(\frac{1}{2}\right)^5$
4) $\left(\frac{1}{4}\right)^3$
5) $5^4 \cdot 5$
6) $2^9 \cdot 2^2 \cdot 2^3$
7) $\frac{4^3}{4^7}$
8) $\frac{3^4}{3}$
9) $(6^2)^2$
10) $(5^3)^2$
11) $\left[\left(\frac{1}{2}\right)^3\right]^4$
12) 3^{-3}
13) $\left(\frac{5}{4}\right)^{-2}$

Compound Interest Formula:

We covered this formula previously in Section 2d. We covered it in that section in relation to technology. Let us recall what we discovered in Section 2d:

Compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$ where

A is the accumulated value, meaning the principal plus the interest you have accumulated after t years.

P is the principal. This is the money you originally invest.

r is the interest rate.

n is the number of times per year the interest is compounded.

t is time in years.

We did this example:

Joyce wants to invest $2000 at a rate of 3%, compounded monthly for 10 years. How much will she have in 10 years?

Let us substitute everything into our formula: $A = 2000\left(1+\frac{.03}{12}\right)^{12(10)}$.

Let us now put this into our scientific (or phone) calculator. We must follow the order of operations:

1) Divide: $\frac{.03}{12} = .0025$

2) Next we add the 1: 1+.0025=1.0025
3) Next we multiply 12 times 10 = 120
4) We raise the 1.0025 to the 120th power: It is usually the x^y key on your calculator. We get 1.349…
5) Now we multiply that by \$2000 to get: \$2,698.71

Let us break down the components of the formula in the context of this example a little further:

A is the accumulated value. This is the final answer we are looking for in this formula.

\$2000 is our P, our principal. This is the amount of our initial investment.

1 is just part of the formula. It is always there in every problem of this form.

.03 is our interest rate, r. .03 is the same as 3%. We must always change our percent to a decimal. Mathematical formulas require numbers.

12 is our n. This is the number of times, per year, the interest is compounded. Since it is compounded monthly, that is 12 times per year.

10 is t, the number of years we will compound the interest. t will always be in years for this formula.

Note: 12 appears again in the exponent. This is the same n as we have in the denominator of the formula. They will always be the same. This seems to confuse many MAT 120 students.

Exercise Set 2:

For the following set of exercises, identify the following: P, r, n, t:

1) $\$1500\left(1+\frac{.05}{4}\right)^{4\cdot 20}$
2) $\$2500\left(1+\frac{.06}{2}\right)^{2\cdot 10}$
3) $\$1275\left(1+\frac{.02}{24}\right)^{24\cdot 2}$
4) $\$500\left(1+\frac{.055}{6}\right)^{6\cdot 25}$
5) $\$150\left(1+\frac{.10}{12}\right)^{12\cdot 30}$

Exercise Set 3:

For the following set of exercises, insert P, r, n, t into our formula. (No need to calculate the final answer).

1) Karen wants to invest \$25,000 at a rate of 2%, compounded weekly for 15 years.

2) Kevin wants to invest \$5,000 at a rate of 4.2%, compounded bimonthly for 25 years.

3) John wants to invest \$35,00 at a rate of 5.5%, compounded semiannually for 10 years.

Continuously Compounded Interest Formula:

We just went through the compound interest formula where n represented how many times per year the interest is compounded. If the interest is continuously compounded we have a different formula. It is:

$$A = P_0 e^{rt}$$

Note: Some books use

$$A = Pe^{rt}$$

Where A is again the accumulated value, P_0 is our principal, e is the base of our exponential formula, r is our interest rate again, and t is time in years. e is an irrational number like π or $\sqrt{2}$. $e = 2.718\ldots$ It is also defined to be as $n \to \infty, \left(1 + \frac{1}{n}\right)^n$. Technically, it is the limit as $n \to \infty$, which requires Calculus to fully understand. Don't worry about the definition of e too much. Just know it is irrational and ≈ 2.718. e as the base of an exponential function occurs naturally in population growth and decay, and in continuously compounded interest. There is an e key in your scientific and graphic calculators.

Example: Let's go back to our original example we have been following: Joyce wants to invest \$2000 at a rate of 3%, compounded monthly for 10 years. How much will she have in 10 years?

Instead of compounding monthly, let us compound this continually. P_0 is \$2000. $r = .03,\ t = 10.$

Therefore, $A = 2000e^{.03 \cdot 10}$. The order of operations is as follows:

1) Multiply .03 times 10 = .30
2) Raise e to the .30 power: 1.35
3) Multiply 1.35 by 2000 = \$2699.72

(Note that we get more money by continuously compounding the interest rather than compounding it monthly).

Exercise Set 4:

Identify P_0, r, t for the exercises in Exercise Set 3, if they were compounded continuously instead of discretely.

Exponential Growth and Decay:

The formula for exponential growth and decay is basically the same one as the continuously compounded interest one: It is:

$$P = P_0 e^{rt}$$

Where P is the final population, P_0 is the initial population, r is the growth or decay rate, and t is time. In this case t may or may not be in years. It depends on the context of the problem, and what you are measuring. r will be positive in a population growth problem, and negative in a decay problem.

Example:

A

Math For Liberal Arts class doubles every 10 days, because all students know it is the best thing happening on Earth! Find the growth rate, r.

To do this problem we observe that P is 2 and P_0 is 1. Why? It doubles, which is always 2 to 1. No matter how many we start with, we end up with twice as much. t is 10 days.

We insert this info into $P = P_0 e^{rt} \rightarrow \quad 2 = e^{r \cdot 10}$. To solve for r, we will require the understanding of logarithms. This is beyond this course. I wanted to present it, just to show how it gets used for future reference. These problems recur often in : Biology, chemistry, mathematics, physics, etc. You will likely encounter them frequently.

Exercise Set 5:

For the following set of exercises, identify P, P_0, t:

1) A certain population triples every 2 years.
2) A substance has a ½ life of 3 hours. (Hint: whatever you start with, your final population is ½).

Example:

What is the final population, if we start with 100, the growth rate is 2%, and the time is 5 hours?

$$P = P_0 e^{rt} \rightarrow \quad P = 100e^{.02 \cdot 5}$$

First we multiply .02 times 5 to get .1. Next, we raise e to the power of .1 to get 1.105. Finally we multiply 1.105 by 100 to get 110.5 This is our final population.

Exercise Set 5:

For the following set of exercises find P_0, r, t:

1) What is the final population, if we start with 2000, the growth rate is 10%, and the time is 2 years?
2) What is the final population, if we start with 1500, the growth rate is 12%, and the time is 17 days?
3) What is the final population, if we start with 100, the decay rate is -2%, and the time is 15 hours?

Exercise Set 6:

Calculate P for the exercise set 5.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 6.3

- Mathematical Excursions...
  Chapter 10.5

Required Topic 6b: Analyze, differentiate and evaluate commonly used formulas.: Analysis of linear models.

MAT-175 Required Topic 6b is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Appropriate use of linear and exponential models.
2. Math Modeling/Real World Applications: Application of linear models
3. Math Modeling/Real World Applications: Construction of linear models

Khan Academy Support: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-linear-equations-functions/8th-linear-functions-modeling/v/exploring-linear-relationships

1. Math Modeling/Real World Applications: Appropriate use of linear and exponential models.

Goal: To be able to recognize if a linear model is being used appropriately.

Definition: A Linear Model is a linear function of the form: $y = mx + b,$ where we have covered this formula extensively. A Linear Model is a real-world application of this function.

Example 1: Joyce is starting a new business. It will cost Joyce \$2500 in start-up costs. These will be fixed. In addition, it will cost her \$25 per unit to produce. Can this scenario be modeled with a linear model? If so, what would it be? Yes, it would be $C(x) = 25x + 2500,\ or\ \ y = 25x + 2500.$ How did we see this? \$2500 is fixed or constant. \$25 per unit is not varying. The only thing that is varying is the number of units produced. This produces a Linear Model.

Example 2: The population of a certain species is doubling every 2 years. If the population starts with 100 animals, how many will it have after 30 years? Will a linear model work? No, this is exponential growth. Anything involving doubling is an exponential growth rate problem.

Exercise Set 1:

For the following set of exercises, determine whether a Linear Model would work for the problem:

1) George is going snowboarding. If his speed is 20 mph, what will his distance be in terms of hours?
2) Mary started a new business. Her revenue will be \$5000 plus \$500 per item sold.
3) Fred is starting his new business. His revenue will be \$600 plus \$20 times the square of a unit sold.
4) A certain chemical has a ½ life of 2 days. If we start with 30g, how much will be left after 3 days?

2. **Math Modeling/Real World Applications: Application of linear models**
3. **Math Modeling/Real World Applications: Construction of linear models**

Goal: To be able to take what we learned above, and construct a linear model. Then, be able to apply the model to specific sets of data.

Example 2: Let us revisit Example 1. Joyce is starting a new business. It will cost Joyce \$2500 in start-up costs. These will be fixed. In addition, it will cost her \$25 per unit to produce. Can this scenario be modeled with a linear model? If so, what would it be? Yes, it would be $C(x) = 25x + 2500,\ or\ \ y = 25x + 2500.$ How did we see this? \$2500 is fixed or constant. \$25 per unit is not varying. The only thing that is varying is the number of units produced. This produces a Linear Model.

The y-intercept, $b,$ will be the constant, or fixed amount. The slope, m will also be constant, but will be a rate of change. We observe that it is \$25/unit, which is a rate.

Here, we were able to determine a linear model would work for this example. We went ahead and constructed the linear model. Now let us evaluate the model given additional information.

Using our Linear Model, how much will it cost Joyce to produce 100 units? We simply substitute 100 in for x and evaluate the cost: $C(x) = 25x + 2500 \rightarrow \quad C(100) = 25(100) + 2500 = \$5000.$

Exercise Set 2:

1) Jonathan is starting a new business. His fixed cost will be \$10,000. In addition, it will cost him \$200 per unit to produce.
   a. Find a linear model that represents the cost of Jonathan's new business.
   b. How much will it cost Jonathan to produce 2000 items?
   c. How much will it cost him to produce 10,000?

2) Karen is starting a business. Her revenue will be \$430 per unit sold.
   a. Find a linear model that represents Karen's revenue.
   b. How much revenue will she receive after selling 400 units?
   c. How much revenue will she receive after selling 1000 units?

3) Mike has a well-established business. He has a fixed cost of \$500. In addition, it will cost him \$50 per unit to produce. His revenue will be \$200 per unit sold.
   a. Find a linear model that represents Mike's cost.
   b. Find a linear model that represents Mike's revenue.
   c. Find a linear model that represents Mike's profit. (Hint: Profit is Revenue minus cost).
   d. Evaluate Mike's cost, revenue, and profit when Mike produces and sells 100 units.

4) Jerry is a skier. He always skis at a rate of 30 mph.

a. Find a linear model that represents Jerry's distance in terms of time, t. (Hint: $d = rt$, which means distance equals rate times time. Recall speed is a rate).
b. Find Jerry's distance when he has skied for one hour.

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 6.10
- Mathematical Excursions...
- Chapter 10.3

Required Topic 6c: Analyze, differentiate and evaluate commonly used formulas.: Review of solving linear equations.

MAT-175 Required Topic 6c is intended to support the following MAT-120 topics:

1. Formal Logic: Inductive and Deductive Reasoning
2. Math Modeling/Real World Applications: Appropriate use of linear and exponential models
3. Math Modeling/Real World Applications: Application of linear models
4. Math Modeling/Real World Applications: Construction of linear models

Khan Academy Support: https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-1

Goal: To be proficient in solving a variety of linear equations to facilitate competency in the MAT 120 topics.

We have already covered linear equations in a variety of ways. In this section, we will review solving them. To accomplish this, we will go through a variety of definitions and properties first.

PROPERTIES:

Properties of Real Numbers:

1) Distributive Property: $a(b+c) = ab + ac$
2) Commutative Property (Addition): $a + b = b + a$
3) Commutative Property (Multiplication): $ab = ba$
4) Associative Property (Addition): $(a+b)+c = a+(b+c)$
5) Associative Property (Multiplication): $(ab)c = a(bc)$

Example 1: Simplify the following expressions by using the above properties to combine like terms:

1) $2x + 5x$: We add like terms to get $7x$
2) $3(x+5)$: We use Property 1): $3x + 5$
3) $2(x+6x)$: We use Property 1) again: We can distribute first, or add like terms first: $2x + 12x = 14x$, or $2(7x) = 14x$.
4) $3 - 2(y-1)$. Let us use Property 1, and then combine like terms: $3 - 2y + 2 = -2y + 5$.

Exercise Set 1:
Simplify the following expressions by using the above properties to combine like terms:

1) $x + 10x$
2) $2x - 12x$
3) $5(x - 1)$
4) $3(2y + 4y)$
5) $-2(x - 4) + 5x$
6) $\frac{1}{2}(2y - 4) + 9y$
7) $3(4x - 7) - 2(3x + 12)$

Solving Linear Equations:

Now that we have reviewed simplifying linear expressions, we will solve linear equations. First, let us review some basic properties:

Addition Property of Equality:

Let a, b, c be real numbers. If $a = b$, then $a + c = b + c$

Example 2: $x - 7 = 2 \rightarrow \quad x - 7 + 7 = 2 + 7 \rightarrow \quad x + 0 = 9 \rightarrow \quad x = 9.$

Subtraction Property of Equality:

Let a, b, c be real numbers. If $a = b$, then $a - c = b - c$

Example 3: $x + 2 = 10 \rightarrow \quad x + 2 - 2 = 10 - 2 \rightarrow \quad x + 0 = 8 \rightarrow \quad x = 8$

Multiplication Property of Equality:

Let a, b, c be real numbers. If $a = b$, then $a \cdot c = b \cdot c$:

Example 4: $\frac{x}{2} = 12 \rightarrow \quad \left(\frac{x}{2}\right) \cdot 2 = 12 \cdot 2 \rightarrow \quad \frac{2x}{2} = 24 \rightarrow \quad 1 \cdot x = 24 \rightarrow \quad x = 24.$

Division Property of Equality:

Let a, b, c be real numbers. If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ provided $c \neq 0$. (Why can't $c = 0$? Functions are undefined where the denominator equals zero. Why? What does division mean? It means the denominator goes into the numerator x times. If the denominator is zero, what value times zero will give you the numerator? Nothing times zero will give you a non-zero numerator. What if numerator and denominator are both zero? Since zero times anything is zero, this would not be unique so is also undefined.)

Example 5: $2x = 12 \rightarrow \quad \frac{2x}{2} = \frac{12}{2} \rightarrow \quad 1 \cdot x = 6 \rightarrow \quad x = 6.$

Steps for Solving a Linear Equation:

1) Use the Distributive Property on each side when necessary.
2) Combine like terms on each side of the equation.
3) Use the Addition and Subtraction Properties.
4) Use the Multiplication and Division Properties.
5) Check your answer.

   Note: If you have fractions, you can multiply both sides of the equation by the lowest common denominator before or after step 1).

Example 6:

Solve the following linear equations:

1) $2x + 3x = 7$: $5x = 7$ (using step 2))→ $x = \frac{7}{5}$ (using step 4)
2) $3(x - 5) = 10$: $3x - 15 = 10$ (using step 1)→ $3x - 15 + 15 = 10 \rightarrow$ $3x = 25$ (using step 3)→ $x = \frac{25}{3}$ (using step 4).
3) $\frac{1}{2}(x - 2) = 3$: $\frac{1}{2} \cdot 2(x - 2) = 3 \cdot 2$ (using step 4 along with clearing fractions)→ $1(x - 2) = 6 \rightarrow$ $x - 2 = 6$ (using step 1)→ $x - 2 + 2 = 6 + 2$ (using step 3)→ $x = 8.$
4) $4(2x - 2) = 3(3x + 4)$: $8x - 8 = 9x + 12$ (using step 1)→ $8x - 8 - 9x = 9x + 12 - 9x \rightarrow$ $8x - 9x - 8 = 9x - 9x + 12$ (using step 4) and Property of Real Numbers #2)→ $-x - 8 = 0 + 12$ (using step 2)→ $-x - 8 = 12 \rightarrow$ $-x - 8 + 8 = 12 + 8$ (using step3)→ $-x = 20 \rightarrow$ $-\frac{x}{-1} = \frac{20}{-1}$ (using step 4)→ $x = -20.$

   Note: We did not check our answers. This is left a an exercise for the student.

Exercise Set 2:

Solve the following linear equations: For the first 3 exercises, list each step you used.

1) $x - 2 = 10$
2) $2x + 5 = 7$
3) $3x - 7 = 2x$
4) $\frac{1}{2}x + 2 = 5$
5) $2(x - 3) = 4x$
6) $\frac{1}{3}(x - 2) = (3x - 1)$
7) $4x(3 - 2) = 5(x + 1)$
8) $10(2x + 1) = 4(x + 7)$
9) $\frac{2}{3}(x - 1) = 3(x + 2)$
10) $\frac{1}{2}(2x + 6) = \frac{1}{3}(x - 2)$
11) $3(x - 2) + 6 = 4(x + 4) - 5$
12) $2(3x - 1) - 2 = 5(10x - 4) - 1$
13) $\frac{1}{2}(x + 7) + 1 = \frac{1}{4}(x - 1) - 1$
14) $7\left(x + \frac{1}{2}\right) = 3x + 2$
15) $3\left(x - \frac{1}{3}\right) + 2 = 2\left(x - \frac{1}{2}\right) - 1$

Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 6.2
- Mathematical Excursions...
- Chapter 9.1

Required Topic 6d: Analyze, differentiate and evaluate commonly used formulas: Review of percent.

MAT-175 Required Topic 6d is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Simple and compound interest
2. Math Modeling/Real World Applications: Applications using percentages such as budgets, sales tax and discounts

Khan Academy Support: https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-percent-problems/v/finding-percentages-example

Goal: To demonstrate proficiency in using percent so that it may be applied to the MAT 120 topics listed above.

Definition: A number or ratio denoted as a fraction of 100.

Steps to find the percent of a fraction:

1) Change the fraction to a decimal by doing long division: Dividing numerator by denominator (or by using a calculator).
2) Multiply the decimal by 100.
3) Add a percent sign, % to the end of the number.

Example 1: Change $\frac{1}{4}$ to a percent: We divide 1 by 4 to get the decimal .25 (step 1). We multiply .25 by 100 to get 25. We add % at the end of 25 to get 25%. $\frac{1}{4}$ is 25%.

Example 2: Change $2\frac{2}{3}$ into a percent: First, we change $2\frac{2}{3}$ from a mixed number into an improper fraction. $2\frac{2}{3} = \frac{8}{3}$. Next, we divide 8 by 3 to get the decimal $2.6\overline{6}$. Now, we multiply $2.6\overline{6}$ by 100 to get 266. Finally, we add % at the end to get 266%.

Example 3: Change .67 into a percent. This one is easier. We simply move the decimal places to the right by two and add the % sign to get 67%.

Exercise Set 1:

Change the following fractions or decimals into percents:

1) .23
2) 1.25

3) $\frac{1}{2}$
4) $\frac{2}{3}$
5) 4.21
6) $\frac{5}{6}$
7) $1\frac{1}{2}$
8) $2\frac{1}{8}$
9) $3\frac{1}{4}$
10) $\frac{7}{8}$

Steps to change a percent to a decimal or fraction:

1) Move the decimal place to the left by two places (means divide by 100), while removing the percent sign. **We now have a decimal.**
2) If we want to go further, to have a fraction, we take the digits after the decimal point and divide by 10,100,1000, etc. depending on the number of digits after the decimal point. It will have the same number of zeroes that the digits we have do, with a 1 in front. Remove the decimal. Reduce the fraction.
3) If we have a mixed number, we turn it into an improper fraction.

Example 4: Let us change 20% into a decimal. We move the decimal point while removing the % sign to get .20.

Example 5: Let us change the decimal in Example 4 into a fraction: $.20 = \frac{20}{100} = \frac{1}{5}$.

Example 6: Let us change $\frac{1}{4}\%$ to a decimal. First, $\frac{1}{4} = .25$, so $\frac{1}{4}\% = .25\%$. Move the decimal place twice to the left while removing the % sign: .0025 is the decimal. $.0025 = \frac{25}{10000} = \frac{1}{400}$.

Exercise Set 2:

Change the following percents into decimals.

1) 25%
2) 33%
3) 1.01%
4) 2%
5) $\frac{1}{4}\%$
6) $2\frac{2}{3}\%$
7) $1\frac{1}{4}\%$

Exercise Set 3:

Change the decimals in Exercise Set 2 into fractions.

Other Types of Percent Problems:

1) **How to find the percent of a number: (Hint: Of means to multiply).**
   a. **Change % into a decimal.**
   b. **Multiply the decimal in a. by the number in the problem.**

Example 7:

What is 10% of $2,000? In this problem we must:

a. Change % into a decimal. $10\% = .10.$
b. Multiply a. by $2000: $.10(2000) = \$200.$

2) **How to find what percent of a number is another number:**
   a. **What can be represented by a variable. Let us use $\boldsymbol{x}$.**
   b. **Of means to multiply, so we get $\boldsymbol{ax = b}$, and we solve for $\boldsymbol{x}$. $\boldsymbol{a}$ and $\boldsymbol{b}$ are constants, the numbers we know. Note: is means =**

Example 8:

What percent of 100 is 5?

We set up an equation: $x \cdot 100 = 5 \rightarrow \quad 100x = 5 \rightarrow \quad x = \frac{5}{100} = .04 = 4\%.$

Exercise Set 4:

1) What is 15% of $3500?
2) What is 3% of $25,000?
3) What is 2.5% of $100,000?
4) What is 1% of $37,000?
5) What percent of 20 is 3?
6) What percent of 1000 is 2.5?
7) What percent of 500 is 20?
8) What percent of 2500 is 15?Additional Textbook Support:

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8^{th} ed.
    - Chapter 11.1
- Mathematical Excursions...
- Chapter 9.3

Required Topic 6e: Analyze, differentiate and evaluate commonly used formulas: Formulas for simple and compound interest.

MAT-175 Required Topic 6e is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Application of basic exponential models.
2. Math Modeling/Real World Applications: Simple and compound interest.
3. Modeling Real-World Financial Problems: Annuities with applications
4. Modeling Real-World Financial Problems: Applications using percentages such as budgets, sales tax and discounts.

Goal: To demonstrate proficiency and fluidity with the formulas for simple and compound interest formulas so they may be applied to modeling in MAT 120. We will save the applications to your MAT 120 class.

Simple Interest Formula: We introduced this formula very briefly in Section 2d. Let us go through this formula carefully, at this time.

Formula: $i = prt$:

1) i is the interest accumulated. (This is the interest only, not the principal plus the interest).
2) p is the principal. This is the initial amount invested.
3) r is the interest rate.
4) t is time in years.

Example 1:

How much interest will you accumulate if you invest \$1000 at a rate of 3% over 2 years if you invest at simple interest? Let us identify p, r, t: $p = \$2000$, $r = 3\% = .03$, $t = 2$. To solve, we simply multiply: $\$2000(.03)(2) = \$120$.

Exercise Set 1:

For the following set of exercises, identify, p, r and t: Do not solve the problem.

1) How much interest will you accumulate if you invest \$25,000 at a rate of 2.5% over 10 years if you invest at simple interest?
2) How much interest will you accumulate if you invest \$3000 at a rate of 5% over 2.5 years if you invest at simple interest?
3) How much interest will you accumulate if you invest \$5000 at a rate of 3.2% over 12 years if you invest at simple interest?

4) How much interest will you accumulate if you invest \$1000 at a rate of 3.65% over 9 years if you invest at simple interest?
5) How much interest will you accumulate if you invest \$150 at a rate of 4% over 7 years if you invest at simple interest?

Exercise Set 2:

Evaluate how much interest you will accumulate for the exercises in Exercise Set 1.

Other Types of Simple Interest Problems:

What if we know the interest accumulated, but don't know one of the other values? We may have to use algebra to solve for one of the other variables in the formula: $i = prt$.

Example 2: You have accumulated \$500 in interest, using simple interest. If you invested p amount initially at 3.2% interest over 10 years, how much (p) did you initially invest? Let us write down what we know: $i = 500,\ \ r = 3.2\% = .032,\ \ t = 10,\ \ p = ?\quad 500 = p(.032)(10)$. p is the only variable left. We will employ the skills we recently learned in solving linear equations. Let us multiply: $(.032)(10) = .32$. Next, we will divide 500 by .32 to get \$1562.50. This was our initial investment.

Example 3: You have accumulated \$640 in interest. If you invested \$10,000 initially over 20 years, what interest rate did you get? ? Let us write down what we know: $i = 640,\ \ r = ?,\ \ t = 20,\ \ p = \$10{,}000\quad 640 = 10{,}000(r)(20)$. r is the only variable left. We will employ the skills we recently learned in solving linear equations. Let us multiply: $(10{,}000)(20) = 200{,}000$. Next, we will divide 640 by 200,000 to get .0032 = .32% This was our interest rate.

Exercise Set 3:

For the following set of exercises, identify, p, r and t: Do not solve the problem.

1) You have accumulated \$510 in interest, using simple interest. If you invested p amount initially at 2.6% interest over 15 years, how much (p) did you initially invest?
2) You have accumulated \$40 in interest. If you invested \$1,000 initially over 2 years, what interest rate did you get?
3) You have accumulated \$610 in interest. If you invested \$15,000 initially over t years at 4% interest, how many years, t, did you invest?

Exercise Set 4:

Write down an equation for the exercises in Exercise Set 3. Do not solve.

Exercise Set 5:

Solve the equations in Exercise Set 4.

Compound Interest Formula:

We briefly introduced the compound interest formula in Section 2d, where we went over how to enter it into a Scientific Calculator. We further developed this formula in Section 6a. Here is what we covered:

Compound interest formula: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where

A is the accumulated value, meaning the principal plus the interest you have accumulated after t years.

P is the principal. This is the money you originally invest.

r is the interest rate.

n is the number of times per year the interest is compounded.

t is time in years.

We did this example:

Joyce wants to invest $2000 at a rate of 3%, compounded monthly for 10 years. How much will she have in 10 years?

Let us substitute everything into our formula: $A = 2000\left(1 + \frac{.03}{12}\right)^{12(10)}$.

Let us now put this into our scientific (or phone) calculator. We must follow the order of operations:

6) Divide: $\frac{.03}{12} = .0025$
7) Next we add the 1: 1+.0025=1.0025
8) Next we multiply 12 times 10 = 120
9) We raise the 1.0025 to the 120th power: It is usually the x^y key on your calculator. We get 1.349…
10) Now we multiply that by $2000 to get: $2,698.71

Let us break down the components of the formula in the context of this example a little further:

A is the accumulated value. This is the final answer we are looking for in this formula.

\$2000 is our P, our principal. This is the amount of our initial investment.

1 is just part of the formula. It is always there in every problem of this form.

.03 is our interest rate, r. .03 is the same as 3%. We must always change our percent to a decimal. Mathematical formulas require numbers.

12 is our n. This is the number of times, per year, the interest is compounded. Since it is compounded monthly, that is 12 times per year.

10 is t, the number of years we will compound the interest. t will always be in years for this formula.

Note: 12 appears again in the exponent. This is the same n as we have in the denominator of the formula. They will always be the same. This seems to confuse many MAT 120 students.

Exercise Set 2:

For the following set of exercises, identify the following: P, r, n, t:

6) $\$1500\left(1+\frac{.05}{4}\right)^{4\cdot 20}$
7) $\$2500\left(1+\frac{.06}{2}\right)^{2\cdot 10}$
8) $\$1275\left(1+\frac{.02}{24}\right)^{24\cdot 2}$
9) $\$500\left(1+\frac{.055}{6}\right)^{6\cdot 25}$
10) $\$150\left(1+\frac{.10}{12}\right)^{12\cdot 30}$

Exercise Set 3:

For the following set of exercises, insert P, r, n, t into our formula. (No need to calculate the final answer).

4) Karen wants to invest \$25,000 at a rate of 2%, compounded weekly for 15 years.

5) Kevin wants to invest \$5,000 at a rate of 4.2%, compounded bimonthly for 25 years.

6) John wants to invest \$35,00 at a rate of 5.5%, compounded semiannually for 10 years.

Continuously Compounded Interest Formula:

We just went through the compound interest formula where n represented how many times per year the interest is compounded. If the interest is continuously compounded we have a different formula. It is:

$$A = P_0 e^{rt}$$

Where A is again the accumulated value, P_0 is our principal, e is the base of our exponential formula, r is our interest rate again, and t is time in years. e is an irrational number like π or $\sqrt{2}$. $e = 2.718 \ldots$ It is also defined to be as $n \to \infty$, $\left(1 + \frac{1}{n}\right)^n$. Technically, it is the limit as $n \to \infty$, which requires Calculus to fully understand. Don't worry about the definition of e too much. Just know it is irrational and ≈ 2.718. e as the base of an exponential function occurs naturally in population growth and decay, and in continuously compounded interest. There is an e key in your scientific and graphic calculators.

Example: Let's go back to our original example we have been following: Joyce wants to invest $2000 at a rate of 3%, compounded monthly for 10 years. How much will she have in 10 years?

Instead of compounding monthly, let us compound this continually. P_0 is $2000. $r = .03,\ t = 10.$

Therefore, $A = 2000e^{.03 \cdot 10}$. The order of operations is as follows:

4) Multiply .03 times 10 = .30
5) Raise e to the .30 power: 1.35
6) Multiply 1.35 by 2000 = $2699.72

(Note that we get more money by continuously compounding the interest rather than compounding it monthly).

Exercise Set 4:

Identify P_0, r, t for the exercises in Exercise Set 3, if they were compounded continuously instead of discretely.

You can do these exercises again, or simply review what you need to.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 11.2, 11.3
- Mathematical Excursions...
- Chapter 11.1, Chapter 11.2

Required Topic 6f: Analyze, differentiate and evaluate commonly used formulas: Formulas for savings plan and loan payments.

MAT-175 Required Topic 6f is intended to support the following MAT-120 topics:

1. Modeling Real-World Financial Problems: Loans with applications
2. Modeling Real-World Financial Problems: Annuities with applications

1. Modeling Real-World Financial Problems: Loans with applications

Khan Academy Support: https://www.khanacademy.org/economics-finance-domain/macroeconomics/monetary-system-topic/macroeconomics-interest-rates-and-the-time-value-of-money/v/introduction-to-present-value

https://www.khanacademy.org/economics-finance-domain/core-finance/interest-tutorial/credit-card-interest/v/annual-percentage-rate-apr-and-effective-apr

Goal: To be comfortable with formulas that apply to loans with applications.

Before we begin with installment loans, let us go over a formula for savings plans:

Present Value:

This is the amount of money you must deposit today in order to receive a certain amount in the future. Perhaps, you want to save for a big vacation in 5 years. Or, perhaps, you need to start saving now to pay or your child's college education in 15 years.

Present Value Formula: $p = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$. Let us observe that this is the compound interest formula, where we have solved for p. In this case we call p, the present value.

p is the present value. This is the money you invest now.

A is the accumulated value, meaning the principal plus the interest you have accumulated after t years.

r is the interest rate.

n is the number of times per year the interest is compounded.

t is time in years.

Example: George and Mary want to send their son to college in 15 years. They need to have $350,000 saved in that amount of time. They can invest the money now in a CD at 3.5% interest, to be compounded monthly.

Let us identify the values, A, r, n, t: (**Note:** We will not solve these problems here. We will leave that for your MAT 120 course).

We note:

$A = \$350{,}000$

$r = 3.5\% = .035$

$n = 12$

$t = 15.$

Exercise Set 4:

For the following set of exercises, identify: A, r, n, t. Do not solve.

1) Donald wants to send his son to college in 10 years. He need to have $200,000 saved in that amount of time. He can invest the money now in a CD at 2.5% interest, to be compounded monthly.
2) Sally wants to send her daughter to college in 15 years. She need to have $250,000 saved in that amount of time. She can invest the money now in a CD at 3.1% interest, to be compounded quarterly.
3) Tom wants to take his family on a vacation in 5 years. He needs to have $10,000 saved in that amount of time. He can invest the money now in a CD at 2.5% interest, to be compounded semi-annually.

Installment Buying:

Installment Buying is a way to repay a loan on a weekly or monthly repayment plan using an Installment Plan. There are two types: One is a Fixed Installment Loan, where you pay a fixed amount over a set number of payment. Example: Vehicle loan. The other is an Open-end Installment Loan like a revolving credit card.

Definitions:

Annual Percentage Rate (APR) is the true interest rate of the loan. This is usually calculated using a table. (This will be left for you MAT 120 class).

Finance Charge: The total amount a borrow must repay for the use of the money: Interest plus fees, insurance, etc.

In addition to tables, you can use a loan calculator on the internet:
https://www.bankrate.com/calculators/mortgages/loan-calculator.aspx

What if I want to pay off the loan early? There will be a reduction in the finance charge. The amount of reduction is called **Unearned Interest.**

Actuarial Method For Unearned Interest:

$$u = \frac{n \cdot P \cdot V}{100 + V}$$

u is the unpaid interest

n is the number of remaining payments including the current one

P is the monthly payment

V is the value from the APR table that corresponds to the APR for the number of remaining payments. (Not including the current one).

Example: Let us assume Tom has a V of 6.45. (We would have to use a table or the internet calculator). Tom has a 3 year loan with monthly payments of \$600. He has paid on the loan for two years. He wants to pay the remaining balance and terminate his loan with his 24th payment.

How much interest will he save by repaying the loan early?

$$u = \frac{n \cdot P \cdot V}{100 + V} = \frac{12 \cdot 600 \cdot 6.45}{100 + 6.45} = \$436.26$$

Exercise Set 5:

For the following set of exercises, identify: $n, p.$ V will be given to you. Put the values into the formula. Do not calculate:

1) Let us assume Terri has a V of 10.50. (We would have to use a table or the internet calculator). Terri has a 5 year loan with monthly payments of \$450. She has paid on the loan for 3 ½ years. She wants to pay the remaining balance and terminate her loan with her 42nd payment.

2) Let us assume Jenny has a V of 9.46. (We would have to use a table or the internet calculator). Jenny has a 4 year loan with monthly payments of $500. She has paid on the loan for two years. She wants to pay the remaining balance and terminate his loan with her 24th payment.

3) Let us assume John has a V of 11.16. (We would have to use a table or the internet calculator). Tom has a 2 year loan with monthly payments of $60. He has paid on the loan for one year. He wants to pay the remaining balance and terminate his loan with his 12th payment.

4) Let us assume Tony has a V of 4.00. (We would have to use a table or the internet calculator). Tony has a 18 month loan with monthly payments of $250. He has paid on the loan for six months. He wants to pay the remaining balance and terminate his loan with his 6h payment.

1. Modeling Real-World Financial Problems: Annuities with applications

Internet Support: https://www.investopedia.com/terms/a/annuity.asp

Goal: To be comfortable with formulas that apply to annuities.

Definition:

Annuity: An annuity is an account in which fixed payments are made to an individual, often as a stream of income for retirees.

Ordinary (or fixed) Annuity: Equal payments are made at regular intervals. The interest is compounded at the end of each interval. The interest rate is fixed.

Ordinary Annuity Formula:

$$A = \frac{p\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$$

A is the accumulated amount.

p is the dollar amount of the payments

r is the interest rate

n is the number of times per year the interest is compounded

t is the number of years.

Example: Joey is depositing \$100 each month into an ordinary annuity that pays 2% interest compounded monthly. Determine the accumulated amount after 20 years.

Let us identify p, r, n, t. $p = \$100,\ r = 2\% = .02,\ n = 12,\ t = 20.$

Next, we will substitute these values into our formula:

$$A = \frac{p\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}} = \frac{100\left[\left(1+\frac{.02}{12}\right)^{12\cdot 20} - 1\right]}{\frac{.02}{12}} = \$29{,}479.68$$

Scientific Calculator: Order of Operations:

Hint: If you have a graphing calculator, you can type it in the way it appears above).

Steps:

1) Divide .02 by 12.
2) Add 1
3) Raise to the 12 times 20 = 240 power.
4) Subtract 1
5) Multiply by 100
6) Divide by $\frac{.02}{12} = .0016\overline{6}$

Exercise Set 5:

For the following set of exercises: Identify p, r, n, t.

1) George is depositing \$150 quarterly into an ordinary annuity that pays 3.1% interest compounded quarterly.
2) John is depositing \$200 each month into an ordinary annuity that pays 2.5% interest compounded monthly.
3) Sally is depositing \$50 twice per year into an ordinary annuity that pays 5% interest compounded semiannually.
4) Suzie is depositing \$100 each month into an ordinary annuity that pays 2.7% interest compounded monthly.

Exercise Set 6:

Substitute p, r, n, t into the Ordinary Annuity Formula in Exercise Set 5. Do not calculate.

Exercise Set 7:

Calculate the Accumulated Amount for the problems in Exercise Set 6.

Sinking Fund:

Definition: A sinking fund is an annuity where the goal is to have a specific amount of money in a specific amount of time.

Sinking Fund Formula:

$$p = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

p is the payment needed to reach the accumulated value.
r is the interest rate.
n is the number of times per year the interest is compounded.
t is the number of years.

Example: Kevin wants to save \$25,000 in 3 years to buy a new car. He is going to invest monthly into a sinking fund that pays 3.2% interest. How much does he need to invest each month to achieve his goal?

Let us first identify, A, r, n, t: $A = \$25{,}000,\ r = 3.2\% = .032,\ n = 12,\ t = 3.$

Next, we will substitute these values into our formula:

$$p = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1} = \frac{25000\left(\frac{.032}{12}\right)}{\left(1+\frac{.032}{12}\right)^{12(3)} - 1} = \$662.69$$

Scientific Calculator: Order of Operations:

Hint: If you have a graphing calculator, you can type it in the way it appears above).

Steps:

1) Divide .032 by 12.
2) Add 1
3) Raise to the 12 times 3 = 36 power.
4) Subtract 1

5) Set this number aside.
6) Divide by $\frac{.032}{12} = .0026\overline{6}$
7) Multiply 6) by 25,000
8) Divide 7) by 5).

Exercise Set 8:

For the following set of exercises: Identify A, r, n, t:

1) Josie wants to save $10,000 in 2 years to buy a used truck. She is going to invest monthly into a sinking fund that pays 2.7% interest. How much does she need to invest each month to achieve her goal?

2) Kyle wants to save $15,000 in 7 years to buy home improvements. He is going to invest monthly into a sinking fund that pays 4.1% interest. How much does he need to invest each month to achieve his goal?

3) Justin wants to save $5,000 in 3 years to buy a new couch. He is going to invest monthly into a sinking fund that pays 2.9% interest. How much does he need to invest each month to achieve his goal?

Exercise Set 9:

Substitute A, r, n, t into the Sinking Fund Formula in Exercise Set 8. Do not calculate.

Exercise Set 10:

Calculate the payment needed to reach the Accumulated Amount for the problems in Exercise Set 9.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 11.6
- Mathematical Excursion...
- Chapter 11.2, Chapter 11.3
- Chapter 11.5

Required Topic 6g: Analyze, differentiate and evaluate commonly used formulas: Geometric formulas, including perimeter/circumference, area, and volume.

MAT-175 Required Topic 6g is intended to support the following MAT-120 topics:

1. Modeling Real-World Financial Problems: Cost estimation using applied geometry

Goal: To have a complete and comfortable relationship with the geometric formulas in order to apply them to real-world problems in MAT 120.

Definitions:

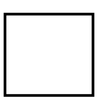
Perimeter: Is the sum of the lengths of all the sides of a two-dimensional figure. Circles have a circumference, which will be covered shortly.

Common Formulas for Perimeters:

Rectangle:

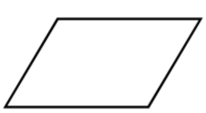
Perimeter, p, is $p = 2l + 2w$, where l is length, and w is width.

Square: Note: A square is a special type of rectangle, where all the sides are equal.



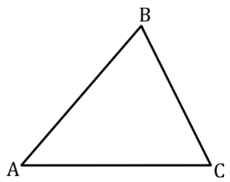
Perimeter, p, is $p = 2l + 2w$, where l is length, and w is width. Since all sides are equal $p = 2l + 2l = 4l$.

Parallelogram:



Perimeter, p, is $p = 2l + 2w$, where l is length, and w is width.

Triangle:

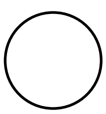


Perimeter, $p = \overrightarrow{AC} + \overrightarrow{BC} + \overrightarrow{AB}$

Trapezoid:

Perimeter, $p =$ side 1 + side 2 +side 3 + side 4.

Circle: A circle is defined to be a set of points equidistant from a fixed point that is its center.



The radius of a circle, r, is a line segment from its center to any point on the circle (the outline). (Note: A circle is an outline. If it is filled in, it is a disk.)

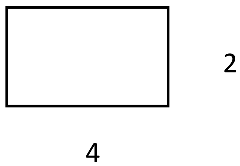
The circumference is the measure of the length of the closed curve. (It is similar to the perimeter of the objects we described previously.) Since it does not have a discrete number of sides it is called a circumference.

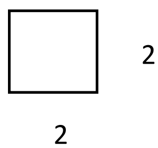
The circumference is $2\pi r$, where $\pi = 3.14 \ldots$ π is an irrational number, as is $\sqrt{2}$ (another example), meaning it cannot be written as a ratio of 2 integers. r is the radius of the circle.

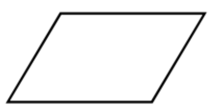
I feel this section is straight-forward enough to skip examples, allowing students to go straight to the exercises.

Exercise Set 1:

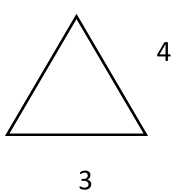
For the following set of exercises, find the perimeter, p:

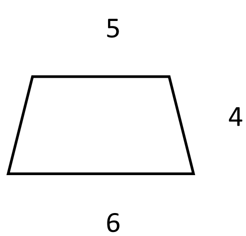
1) 

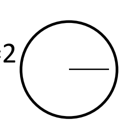
2) 



3) Sides 3 x 4

4) 

5) 

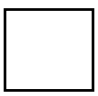
6) r=2 

Area: Is the region within the figure. Its units are always in $(units)^2$. You may be familiar with this concept, e.g., the square footage of your home.

Rectangle:

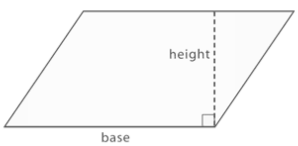
Area, $A = l \cdot w,$ where l is length, and w is width.

Square: Note: A square is a special type of rectangle, where all the sides are equal.



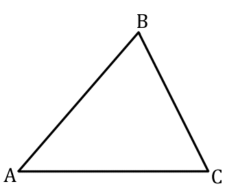
Area, $A = l \cdot w,$ where l is length, and w is width.. Since all sides are equal $A = l^2$.

Parallelogram:



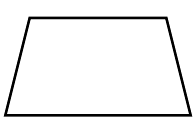
Area, $A = b \cdot h,$ where b is base, and h is height.

Triangle:



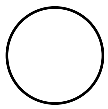
Area, $A = \frac{1}{2} \cdot b \cdot h,$ where b is base, and h is height.

Trapezoid:



Area, $A = \frac{1}{2} \cdot h(b_1 + b_2)$, where b_1 is one base, b_2 is the other base, and h is height.

Circle: A circle is defined to be a set of points equidistant from a fixed point that is its center.



Area, $A = \pi r^2$, where r is the radius of the circle.

Exercise Set 2:

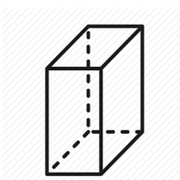
Find the area, A, for the exercises in Exercise Set 1:
Note: The height for #3) and #4) is 3.7, and for #5 is 3.9.

Volume and Surface Area:

Volume: Is the capacity of a 3-dimensional figure. Its units will always be $(units)^3$.

Surface Area: Is the sum of the areas of each side of the figure. The units will be in $(units)^2$.

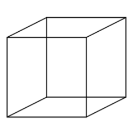
Rectangular Box (or Rectangular Solid):



Volume, $V = lwh$, where l is the length, w is the width, and h is the height.

Surface area, $SA = 2lw + 2wh + 2lh$

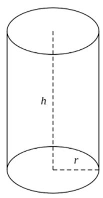
Cube: This is a special rectangular solid where all 3 sides are equal.



Volume, $V = l^3$, where l is the length of each side..

Surface area, $SA = 6l^2$

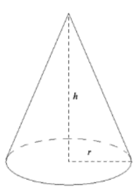
Cylinder:



Volume, $V = \pi r^2 h,$ where r is the radius of the circle that forms the top and bottom of the cylinder, and h is the height of the cylinder.

Surface Area, $SA = 2\pi rh + 2\pi r^2$, where r, h are as indicated above.

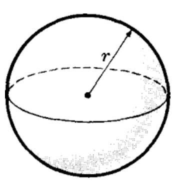
Cone:



Volume $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base of the cone, and h is its height.

Surface Area, $SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}$, where r, h are as indicated above.

Sphere:



Volume: $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

Surface Area: $SA = 4\pi r^2$, where r is as above.

Example: Find the volume and surface area for the following:

1) Rectangular box with $l = 3\ ft, w = 1\ ft, h = 2\ ft.$
   Volume, $V = lwh,$ where l is the length, w is the width, and h is the height. This is: $V = lwh = 3 \cdot 1 \cdot 2 = 6\ ft^3$

   Surface area, $SA = 2lw + 2wh + 2lh = 2 \cdot 3 \cdot 1 + 2 \cdot 1 \cdot 2 + 2 \cdot 3 \cdot 2 = 6 + 4 + 12 = 22\ ft^2.$

2) Cylinder with radius 3 in, and height 10 in.

   Volume, $V = \pi r^2 h,$ where r is the radius of the circle that forms the top and bottom of the cylinder, and h is the height of the cylinder. $V = \pi r^2 h = \pi \cdot 3^2 \cdot 10 = 90\pi\ in^3$

   Surface Area, $SA = 2\pi rh + 2\pi r^2$, where r, h are as indicated above.

   $SA = 2\pi rh + 2\pi r^2 = 2\pi \cdot 3 \cdot 10 + 2\pi \cdot 3^2 = 60\pi + 18\pi = 78\pi\ in^2.$

3) Cone with radius 2 cm, and height 8 cm.

   Volume $V = \frac{1}{3}\pi r^2 h,$ where r is the radius of the base of the cone, and h is its height.

   $V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi \cdot 2^2 \cdot 8 = \dfrac{32}{3}\pi\ cm^3$

   Surface Area, $SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}$, where r, h are as indicated above.

   $$SA = \pi r^2 + \pi r\sqrt{r^2 + h^2} = \pi \cdot 2^2 + \pi \cdot 2\sqrt{2^2 + 8^2} = 4\pi + 2\pi\sqrt{68} = 4\pi + 2\pi \cdot 2\sqrt{17} =$$

   $$4\pi + 4\pi\sqrt{17}\ cm^2.$$

4) Sphere with radius 5 m.

   Volume: $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{500}{3}\pi\ m^3.$

   Surface Area: $SA = 4\pi r^2$, where r is as above. $SA = 4\pi r^2 = 4\pi \cdot 5^2 = 100\pi\ m^2.$

Exercise Set 3:

Find the volume and surface area for the following:

1) Rectangular box with $l = 6\ ft, w = 3\ ft, h = 5\ ft.$

2) Cube with sides $l = 7\ cm.$

3) Cylinder with radius 6 m, and height 20 m.

4) Cone with radius 3 cm, and height 11 cm.

5) Sphere with radius 15 in.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 9.3, 9.4.
- Mathematical Excursion...
  Chapter 7.3

Required Topic 6h: Analyze, differentiate and evaluate commonly used formulas: Formulas used in probability.

MAT-175 Required Topic 6h is intended to support the following MAT-120 topics:

1. Probability and Statistics: Theoretical and empirical probability

Goal: To be able to use and differentiate between the different types of formulas used in probability in order to demonstrate proficiency in MAT 120 probability sections.

Definitions:

Empirical Probability:

Experiment: A controlled operation with a set of results.

Outcomes: The possible results of an experiment.

Event: A sub-collection of outcomes of an experiment.

Empirical Probability Formula: $P(E) = \frac{\#\ times\ event\ E\ occurs}{Total\ \#\ times\ experiment\ is\ performed}$.

Where $P(E)$ is the relative frequency of the experiment.

(In your MAT 120 class you will do experiments). Let us move onto Theoretical Probability:

Theoretical Probability:

$$P(E) = \frac{\#\ Outcomes\ favorable\ to\ E}{Total\ \#\ possible\ outcomes},$$

Where $P(E)$ is the probability that an event E will occur.

Note: $\mathbf{0 \leq P(E) \leq 1}$. You can have a probability of zero. Example: Determine the probability that the sun will rise in the west tomorrow. You can have a probability of one. Example: Determine the probability that the sun will rise in the east tomorrow. Probabilities cannot be < 0 or > 1.

Note: Many of the problems in MAT120 involve dice or cards. Let us review features of each for students who may be unfamiliar, or as a review.

Dice:

Dice are cubes with 6 sides. Each side has 1,2,3,4,5 or 6 dots on it. The number of dots only occur once.

Cards: Regular playing cards:

1) 52 cards in a deck
2) 13 cards of each suit
3) Suits are: Spades, Clubs, Diamonds, Hearts (There are 4).
4) Cards go Ace,2,3,4,5,6,7,8,9,10,Jack,Queen,King. (There are 4 of each).
5) Spades and Clubs are black, while Diamonds and Hearts are red.

Example:

1) A single die is rolled. Determine the probability of rolling a 6.
   $P(E) = \frac{\# \, Outcomes \, favorable \, to \, E}{Total \, \# \, possible \, outcomes} = \frac{1}{6}$. We calculated this because there is only 1 die with 6 dots, and there are 6 possibilities.

2) A single die is rolled. Determine the probability of rolling an even #.
   $P(E) = \frac{\# \, Outcomes \, favorable \, to \, E}{Total \, \# \, possible \, outcomes} = \frac{3}{6} = \frac{1}{2}$. We have 3 even numbers, and 6 total numbers, or ½ are even.

3) One card from a standard deck is selected. Determine the probability of getting a 2.
   $P(E) = \frac{\# \, Outcomes \, favorable \, to \, E}{Total \, \# \, possible \, outcomes} = \frac{4}{52} = \frac{1}{13}$. We have four 2's, and we have 52 total cards.

4) One card from a standard deck is selected. Determine the probability of getting a spade.
   $P(E) = \frac{\# \, Outcomes \, favorable \, to \, E}{Total \, \# \, possible \, outcomes} = \frac{13}{52} = \frac{1}{4}$. We have 13 spades, and 52 total cards.

Exercise Set 1:

List the parts of the problem. Do **not** find the probability yet.

1) A single die is rolled. You will soon determine the probability of rolling a 3. R record how many 3's you can have. Record the total number of possibilities on a die.
2) A single die is rolled. You will soon determine the probability of rolling an odd number. Record how many odd numbers you can have. Record the total number of possibilities on a die.
3) A single die is rolled. You will soon determine the probability of rolling a number greater than 2. Record how many numbers greater than 2 that you can have. Record the total number of possibilities on a die.
4) A single die is rolled. You will soon determine the probability of rolling a number less than or equal to 6. Record how many numbers less than or equal to 6 that you can have. Record the total number of possibilities on a die.

5) One card from a standard deck is selected. You will soon determine the probability of getting an ace. Record how many aces are in a deck. Record the total number of cards in a deck.
6) One card from a standard deck is selected. You will soon determine the probability of getting a red card. Record how many red cards are in a deck. Record the total number of cards in a deck.
7) One card from a standard deck is selected. You will soon determine the probability of getting the queen of spades. Record how many queens of spades are in a deck. Record the total number of cards in a deck.
8) One card from a standard deck is selected. You will soon determine the probability of getting a heart. Record how many hearts are in a deck. Record the total number of cards in a deck.

Exercise Set 2:

Find the probabilities in Exercise Set 1 by dividing the answers in each problem.

Odds:

First Note: Many MAT 120 students get probabilities and odds mixed up. We need to carefully differentiate between the two.

We have Odds Against an Event, and Odds in Favor of an Event. One is the reciprocal of the other.

We observe odds frequently in relation to gambling, like horse races and in casinos.

Odds Against an Event: $\frac{P(failure)}{P(success)}$. This ends up being a fraction, that we write $P(failure){:}\,P(success)$. Note that we have a ratio of probabilities. This differentiates odds from probabilities.

Odds in Favor of an event: $\frac{P(success)}{P(failure)}$. Note that this is simply the reciprocal of the odds against an event occurring.

When you calculate one, you can simply "flip" it to get the other.

Example: Determine the odds against, and then the odds in favor of rolling a 2 when a single die is rolled.

Odds Against: $\frac{5/6}{1/6} = \frac{5}{1} = 5{:}\,1$. Let us break down what we did. In the numerator, we calculated the probability we would not roll a 2. There is only one 2, so there are 5 numbers that are **not** 2. So we get 5/6, since there are 6 total possibilities. In the denominator, we calculated the probability we would roll a 2. There is only one 2, and 6 total numbers to get 1/6. We simplify the fraction. Since it is a complex fraction, and both denominators are the same, they cancel. This will happen every time. We can always write the fraction as the top number in the numerator, divided by the top number in the denominator. We then rewrite the ratio with a colon.

Odds in favor: We simply "flip" our answer to get $\frac{1}{5} = 1{:}\,5.$

Example: A single card is chosen from a standard deck of cards. Determine the odds against, and in favor of getting a diamond.

Odds Against: $\frac{39/52}{13/52} = \frac{39}{13} = \frac{3}{1} = 3{:}\,1.$

Odds in favor: We simply "flip" our answer to get $\frac{3}{1} = 3{:}\,1.$

Exercise Set 3:

Determine the odds against, and then the odds in favor of the following:

Note: You will recognize the similarity of these problems to those in Exercise Set 1. (They require using a probability you found in Exercise 2, as well as its reciprocal). You can use that to your advantage, and shorten your effort.

Our hope is that by repeating this exercise in determining Odds instead of Probabilities, it will reinforce the difference between Odds and Probabilities.

1) A single die is rolled. Determine the odds against, and then in favor, of rolling a 3.
2) A single die is rolled. Determine the odds against, and then in favor of rolling an odd number.
3) A single die is rolled. Determine the odds against, and then in favor of rolling a number greater than 2.
4) A single die is rolled. Determine the odds against, and then in favor of rolling a number less than or equal to 6.
5) One card from a standard deck is selected. Determine the odds against, and then in favor of getting an ace.
6) One card from a standard deck is selected. Determine the odds against, and then in favor of getting a red card.
7) One card from a standard deck is selected. Determine the odds against, and then in favor of getting the queen of spades.
8) One card from a standard deck is selected. Determine the odds against, and then in favor of getting a heart.

Expected Value:

Expected value is used to determine results you would expect of an experiment over a long term.

Expected Value Formula: $E = P_1 \cdot A_1 + P_2 \cdot A_2 + P_3 \cdot A_3 + \cdots + P_n \cdot A_n.$

Where P_1 is the probability the first event will occur.
A_1 is the net amount won or lost if the first event occurs.
P_2, A_2 correlate to the second event
Etc.

(Note: n just means it is the last event. It represents the total number of events).

Example:

John is taking a multiple-choice test. Each question has 4 possible answers. You are penalized for guessing. (Which occurs in many standardized tests). He will receive 5 points for each correct answer, and will lose 1 point for an incorrect answer. He will receive a 0 for leaving it blank.

Let us determine the expected value for John guessing an answer. There are only 2 events: 1) Guessing correctly, and 2) Guessing incorrectly: We will then decide if John should guess or not.

$E = P_1 \cdot A_1 + P_2 \cdot A_2.$

Let us determine the probabilities of P_1, P_2. Guessing correctly: $P_1 = \frac{1}{4}$. Guessing incorrectly: $P_2 = \frac{3}{4}$.
What about A_1, A_2? What does John win if he guessing correctly? He wins 5 pts. What does he win if he guessing incorrectly? He loses 1 point, or -1.

Therefore, $E = \frac{1}{4} \cdot 5 + \frac{3}{4} \cdot (-1) = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$. Since $\frac{1}{2}$ is a positive number, it is a net win for John. He should go ahead and guess.

Exercise Set 4:

1) Mary taking a multiple-choice test. Each question has 5 possible answers. You are penalized for guessing. (Which occurs in many standardized tests). She will receive 4 points for each correct answer, and will lose 1 point for an incorrect answer. She will receive a 0 for leaving it blank.

   a. Decide how many events there are.
   b. Find the probability of each event.
   c. Determine the amount won or lost for each event.
   d. Determine the expected value.
   e. Decide if Mary should guess.

2) Suzie is playing a game with Tony. Suzie picks a card from a deck. If she selects a queen, Tony gives her \$20. If she does not, she gives Tony \$5.

   a. Decide how many events there are for Suzie.
   b. Find the probability of each event for Suzie.
   c. Determine the amount won or lost for each event for Suzie.
   d. Determine the expected value for Suzie..
   e. Decide how many events there are for Tony.
   f. Find the probability of each event for Tony.
   g. Determine the amount won or lost for each event for Tony.
   h. Determine the expected value for Tony..
   i. Decide who has the advantage.

AND and OR Problems:

These problems involve more than one possible outcome. (We will stick to two possibilities).

Loosely, in probability, we can remember "and" means to multiply, and "or" means to add. There is a caveat for "or".

Formulas:

AND:
$P(A \text{ and } B) = P(A) \cdot P(B)$

OR:
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$

Note: Recall what we learned during set theory. "And" represents the intersection of 2 sets (the overlapping region.). The reason we subtract if off, is so that it is not counted twice. Recall that when we constructed Venn Diagrams, we kept subtracting regions, so as not to over count our total.

OR Examples:

1) A single die is rolled. Determine the probability of rolling a 1 or a 6. We must determine the probability of each. The probability of rolling a 1, is 1/6. The probability of rolling a 6, is also 1/6. Is there an overlapping region? This means is there a possibility of it being both a 1 and a 6? No. There is no intersection, so no subtraction here. (We call these mutually exclusive events).

$$P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}.$$

2) A single die is rolled. Determine the probability of rolling a 2 or an even number. We must determine the probability of each. The probability of rolling a 2, is 1/6. The probability of rolling an even number is 3/6 = 1/2. Is there an overlapping region? Yes. There is one number that is a 2, and is also even: It is the number 2. Since there is only 1, the probability of the intersection is 1/6.

$$P(A) + P(B) - P(A \text{ and } B) = \frac{1}{6} + \frac{3}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}.$$

3) One card is selected at random from a standard deck. Determine the probability of selecting a black card or a 3. The probability of selecting a black card is 26/52. The probability of selecting a 3 is 4/52. What is the intersection? This means are there any 3's that are also black? Yes, two. So P (A and B) = 2/52.

$$P(A) + P(B) - P(A \text{ and } B) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}.$$

4) One card is selected at random from a standard deck. Determine the probability of selecting a card less than 4 or a diamond. (In this exercise, Ace is considered low). The probability of selecting a card less than 4 is 12/52. Why? Ace is low, so there is Ace, two and three. There are 4 of each, which makes 12. The probability of selecting a diamond is 13/52. What is the intersection? This means are there any numbers less than 4 that are also diamonds? Yes, there are 3. There is one "Ace of diamonds", one "two of diamonds", and one "three of diamonds". So P (A and B) = 3/52.

$$P(A) + P(B) - P(A \text{ and } B) = \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}.$$

Exercise Set 5:

1) A single die is rolled. Determine the probability of rolling a 2 or a 4.
   a. Determine the probability of rolling a 2.
   b. Determine the probability of rolling a 4.
   c. Determine whether there is an intersection (overlapping region).
   d. If there is an intersection, determine its probability.
   e. Substitute a. through d. into the formula: $P(A) + P(B) - P(A \text{ and } B)$
   f. Determine the probability of the problem.

2) A single die is rolled. Determine the probability of rolling a 3 or a number greater than 2.
   a. Determine the probability of rolling a 3.
   b. Determine the probability of rolling a number greater than 2.
   c. Determine whether there is an intersection (overlapping region).
   d. If there is an intersection, determine its probability.
   e. Substitute a. through d. into the formula: $P(A) + P(B) - P(A \text{ and } B)$
   f. Determine the probability of the problem.

3) One card is selected at random from a standard deck. Determine the probability of selecting a heart or a queen.

a. Determine the probability of selecting a heart.
b. Determine the probability of selecting a queen.
c. Determine whether there is an intersection (overlapping region).
d. If there is an intersection, determine its probability.
e. Substitute a. through d. into the formula: $P(A) + P(B) - P(A\ and\ B)$
f. Determine the probability of the problem.

4) One card is selected at random from a standard deck. Determine the probability of selecting a red card or a spade.
   a. Determine the probability of selecting a red card.
   b. Determine the probability of selecting a spade.
   c. Determine whether there is an intersection (overlapping region).
   d. If there is an intersection, determine its probability.
   e. Substitute a. through d. into the formula: $P(A) + P(B) - P(A\ and\ B)$
   f. Determine the probability of the problem.

AND Problems:

Recall: "And" means to multiply.
$P(A\ and\ B) = P(A) \cdot P(B)$

Many MAT 120 students mix up AND and OR problems. <u>Note,</u> in the AND formula, there is no subtraction! Remember we are going to MULTIPLY!

Example:

Two cards are selected at random from a standard deck of cards. (We mean one at a time). Determine the probability the first is 2, and the second is a queen.

We are going to determine this is 2 different ways:
1) With Replacement
2) Without Replacement

What does that mean?
- With replacement means you put the card back in the deck before you select the 2^{nd} card.
- Without replacement means you set the card aside before you select the 2^{nd} card.

1) With Replacement: The probability of getting a 2 is $4/52$ (since there are 4 2's in a deck). We put the 2 back into the deck. The probability of getting a queen is $4/52$. We multiply:
   $P(A\ and\ B) = P(A) \cdot P(B) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169}$.
2) Without Replacement. The probability of getting a 2 is $4/52$. We leave the 2 out of the deck. There are 51 cards left. The probability of the first event is still $4/52$. This never changes. We always start at the same place. The probability of selecting a queen is $4/51$. Why? There are still 4 queens in the deck. We removed a 2, not a queen. Since the 2 is gone, there are only 51 cards left.

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{4}{52} \cdot \frac{4}{51} = \frac{1}{13} \cdot \frac{4}{51} = \frac{4}{663}.$

Example:

Two cards are selected at random from a standard deck of cards. (We mean one at a time). Determine the probability they are both clubs.

Again, we are going to determine this is 2 different ways:

1) With Replacement
2) Without Replacement

1) With Replacement: The probability of getting a club is 13/52 = 1/4. We put the club back into the deck. The probability of getting a club is 13/52=1/4 . We multiply:
   $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}.$
2) Without Replacement. The probability of getting a club is 13/52 = 1/4. We leave the club out of the deck. There are 12 clubs left. There are 51 cards left. The probability of the first event is still 1/4. . The probability of selecting a club the second time is 12/51 = 4/17. Why? There are only 12 clubs left in the deck. Since a card is gone, there are only 51 cards left.

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{4}{17} = \frac{1}{17}.$

Exercise Set 5:

1) Two cards are selected at random from a standard deck of cards. (We mean one at a time). Determine the probability the first is an Ace, and the second is a spade?

   You are going to determine this is 2 different ways:
   1*) With Replacement
   2*) Without Replacement

   a. What is the probability of the first event occurring?
   b. What is the probability of the 2^{nd} event occurring if you put the card back?
   c. How many aces are left after removing the Ace?
   d. How many total cards are left after removing the Ace?
   e. What is the probability of the 2^{nd} event occurring if you leave the card out?
   f. Determine the probabilities in 1*) and 2*).

2) Two cards are selected at random from a standard deck of cards. (We mean one at a time). Determine the probability they are both kings?

   You are going to determine this is 2 different ways:

1*) With Replacement
2*) Without Replacement

a. What is the probability of the first event occurring?
b. What is the probability of the 2nd event occurring if you put the card back?
c. How many kings are left after removing the king?
d. How many total cards are left after removing the king?
e. What is the probability of the 2nd event occurring if you leave the card out?
f. Determine the probabilities in 1*) and 2*).

3) Two cards are selected at random from a standard deck of cards. (We mean one at a time). Determine the probability the first is a black card, and the second is a club?

You are going to determine this is 2 different ways:
1*) With Replacement
2*) Without Replacement

g. What is the probability of the first event occurring?
h. What is the probability of the 2nd event occurring if you put the card back?
i. How many black cards are left after removing the one black card?
j. How many total cards are left after removing the black card?
k. What is the probability of the 2nd event occurring if you leave the card out?
l. Determine the probabilities in 1*) and 2*).

Conditional Probability:

Conditional Probability is the probability of a 2nd event, E_2, occurring given a 1st event, E_1, has occurred or will occur. We call this $P(E_2|E_1)$. This reads the probability of E_2 occurring given E_1 has or will occur.

Formula: $P(E_2 \big| E_1) = \frac{n(E_1 and\ E_2)}{n(E_1)}$. This formula frequently confuses MAT 120 students. They often don't understand what it means, and they especially tend to get confused by the denominator.

The numerator is the intersection (or overlapping region again). It represents the number of ways both E_1 and E_2 can occur simultaneously. The denominator is only the number of ways E_1 can occur. **Be sure to understand E_1 represents the GIVEN.**

Example:

Test Scores	**Mr. Jones' Class**	**Mrs. Jones' Class**	**Total**
60-75	7	3	10
76-90	12	15	27
91 and above	5	6	11
Total	24	24	48

1) Determine the probability you were in Mr. Jones' class given you got a 79.

$$P(E_2 \mid E_1) = \frac{n(E_1 and\ E_2)}{n(E_1)}$$

We look for the intersection of Mr. Jones' class and scoring a 79. We see it is 12. We look for the **GIVEN:** We read the given is a score of 79. How many students in total got a score in the range it is in? 27.

Therefore: $\frac{n(E_1 and\ E_2)}{n(E_1)} = \frac{12}{27} = \frac{4}{9}$.

2) Determine the probability you got a 100, given you were in Mrs. Jones' class.

$$P(E_2 \mid E_1) = \frac{n(E_1 and\ E_2)}{n(E_1)}$$

We look for the intersection of scoring a 100, and being in Mrs. Jones' class. We see it is 6. We look for the **GIVEN:** We read the given is being in Mrs. Jones' class. How many students in total were in Mrs. Jones' class? 24.

Therefore: $\frac{n(E_1 and\ E_2)}{n(E_1)} = \frac{6}{24} = \frac{1}{4}$.

Exercise Set 6:

Small Town Salary	**Lives on Hill**	**Lives in Valley**	**Total**
\$20,000-\$40,000	110	535	645
\$45,000-\$90,000	207	150	357
\$91,000 and above	1000	27	1027
Total	1317	712	2029

1) We will determine the probability that you make \$35,000, given you live in the hill.
   a. First, how many people have a salary in that range that also live on the hill?
   b. What is the given in this problem?
   c. Where does the given go? What will be the denominator?
   d. Calculate the probability.

2) We will determine the probability you live in the valley, given you make \$100,000.
   a. First, how many people have a salary in that range that also live in the valley?
   b. What is the given in this problem?
   c. Where does the given go? What will be the denominator?
   d. Calculate the probability.

3) We will determine the probability you make \$50,000, given you live in the valley.
   a. First, how many people have a salary in that range that also live in the valley?
   b. What is the given in this problem?
   c. Where does the given go? What will be the denominator?

d. Calculate the probability.

4) We will determine the probability you live on the hill, given you make $60,000.
   a. First, how many people have a salary in that range that also live on the hill?
   b. What is the given in this problem?
   c. Where does the given go? What will be the denominator?
   d. Calculate the probability.

5) We will determine the probability you make $21,000, given you live on the hill.
   a. First, how many people have a salary in that range that also live on the hill?
   b. What is the given in this problem?
   c. Where does the given go? What will be the denominator?
   d. Calculate the probability.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 12.1-12.4, 12.6-12.7.
- Mathematical Excursion...
  Chapter 12.3-12.6

Required Topic 7a: Construct and analyze statistical graphs: Understanding how to read a variety of statistical graphs.

MAT-175 Required Topic 7a is intended to support the following MAT-120 topics:

1. Probability and Statistics: Table and chart analysis
2. Probability and Statistics: Normal distribution

Khan Academy Support https://www.khanacademy.org/math/ap-statistics/quantitative-data-ap/histograms-stem-leaf/v/interpreting-histograms

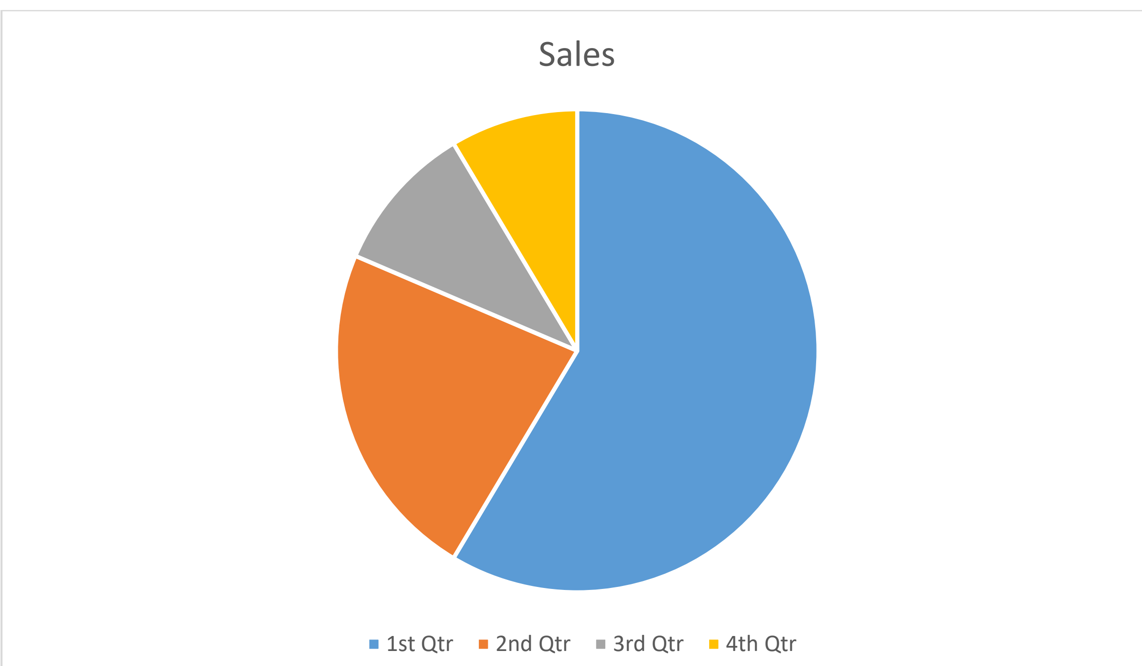
1. Probability and Statistics: Table and chart analysis

Goal: To be able to read and interpret a variety of Statistical graphs.

Circle Graphs (Or Pie Charts):

These are used to compare different parts of a whole set of data to the whole set.

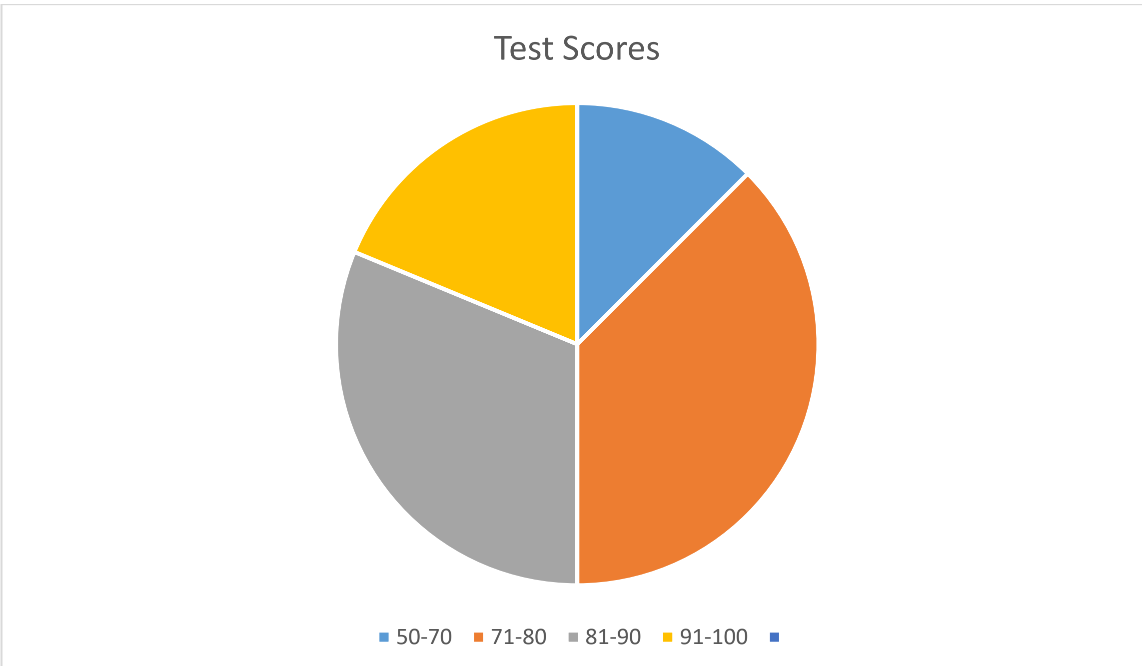
Example:



1) Which quarter had the highest number of sales? The 3rd quarter. It is the quarter in blue, and it has the greatest area inside the circle.

2) Which quarter had the lowest number of sales? The 2nd quarter. It is the smallest area.

Exercise Set 1:



1) Which test score group had the most students?
2) Which test score group had the least amount of students?
3) Put the test score average groups in order from least amount of students to most students.

Histogram:

We constructed histograms in section 5a. Let us review what we did.
Let us review how to graph a **Histogram:**

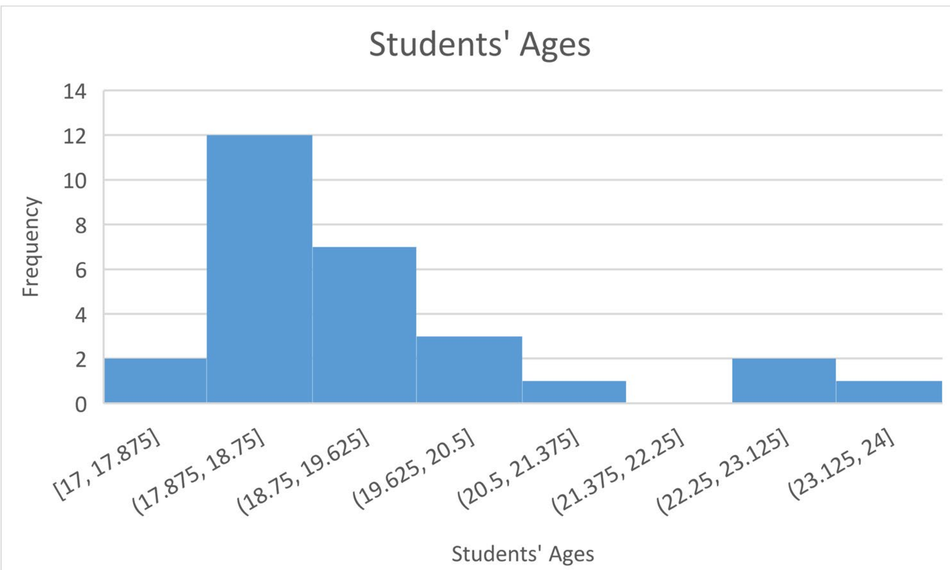
A Histogram is a statistical graph that illustrates a frequency distribution. This is sometimes referred to as a bar graph. Along the x-axis (the horizontal axis), we plot the observed values (or group of values: classes) given. Along the y-axis we plot the frequency of the value(s), i.e. the number of time said value(s) occur.

Example:

Observe the following frequency distribution:

Ages of Students	# of Students of that age
17	2
18	12
19	7
20	3
21	1
22	0
23	2
24	1

Let us make a Histogram for the following set of data;



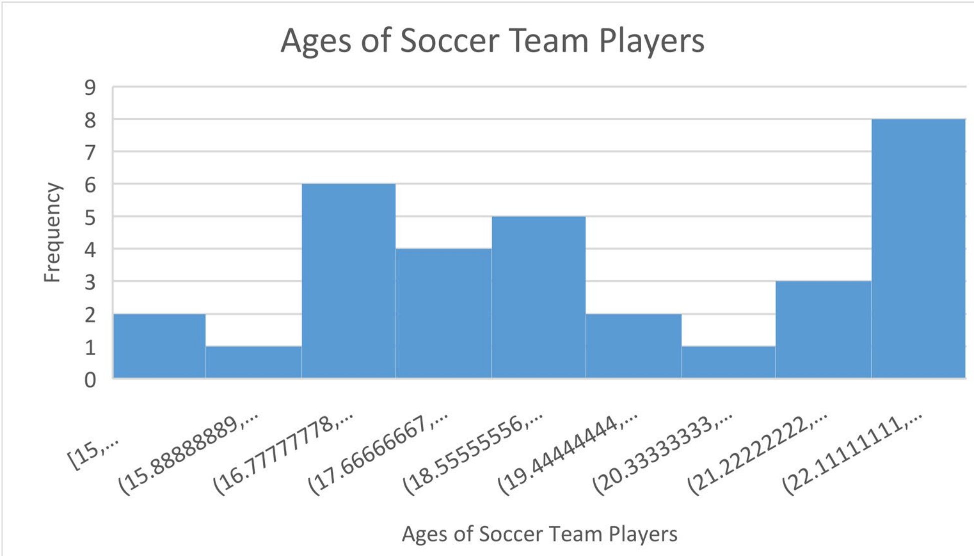
We then did some exercises constructing different histograms.

In this section, we will concentrate on interpreting the data from the graph.

For the above graph: (**We will use the graph instead of the frequency distribution table**.)

1) What student age group had the highest number of students? 18 years old
2) What student age group had the lowest number of students? 22 years old
3) How many total students were there? 28
4) Were there any age groups that had the same number of students? Yes. 17 and 23 both had 2. And 21 and 24 both had 1.

Exercise Set 2:

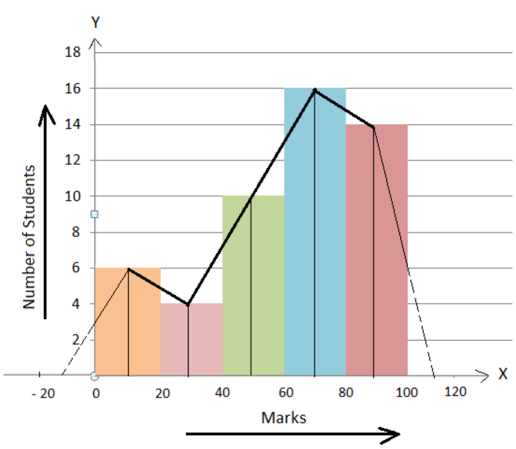


1) What age group (groups) had the largest number of students?
2) What age group (groups) had the least number of students?
3) How many students were age 18?
4) How many students were age 23?

Frequency Polygons: A frequency polygon is a line graph that connects by a dot at the center of each bar. It has the same scale as the histogram.

Example:

Here is a frequency polygon layered on top of a histogram:



Exercise Set 3:

Using the frequency polygon above find the following:

1) How many students got a 10%?
2) How many students got a 30%?
3) What score did the most students receive?

Stem and Leaf Displays:

These are a more accurate way to view data by seeing the actual values. Sometimes histograms and frequency polygons have the data grouped.

The stem is on the left, and the leaves are on the right. The number on the stem will be the first part of a number. The leaf usually contains the last digit of the number. Each number on the leaf comprises a different number.

Example:

The following stem and leaf display contains the ages of the students in the class.

STEM	LEAVES
1	7 8 9
2	0 1 2 3 4 6 8
3	0 1 3

The first row contains ages 17, 18, 19
The second row contains ages 20, 21, 22, 23, 24, 26, 28
The third row contains ages 30, 31, 33.

Exercise Set 4:

The following stem and leaf display contains the weekly salary for an employee for 4 months. The first row is the salaries for month 1, the second row is for month 2, etc. The first number in row one is the salary for week 1 in month 1, the second number is the salary for week 2 in month 1, etc. The second row is the salaries for month 2, and so on.

STEM	LEAVES
60	5 0 2 7
70	5 2 8 3
60	1 5 6 4
50	2 0 0 3

1) How much did the employee earn during month 2, and week 2 of that month?
2) How much did the employee earn during month 4, and week 3 of that month?
3) How much did the employee earn during month 1, and week 1 of that month?
4) How much did the employee earn during month 3, and week 4 of that month?
5) How much did the employee earn during month 2, and week 1 of that month?
6) How much did the employee earn during month 1 and week 3 of that month?
7) How much did the employee earn during month 3, and week 2 of that month?

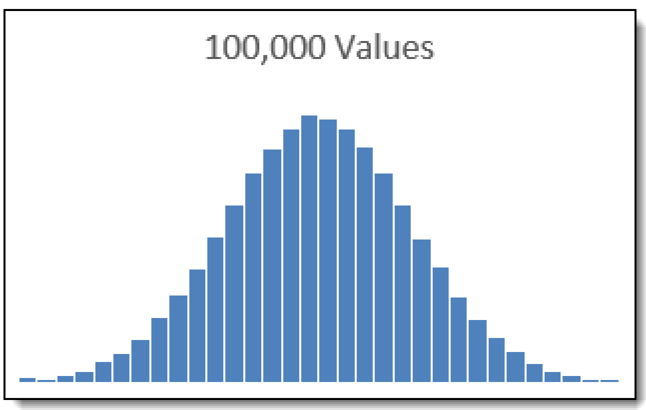
2. **Probability and Statistics: Normal distribution**

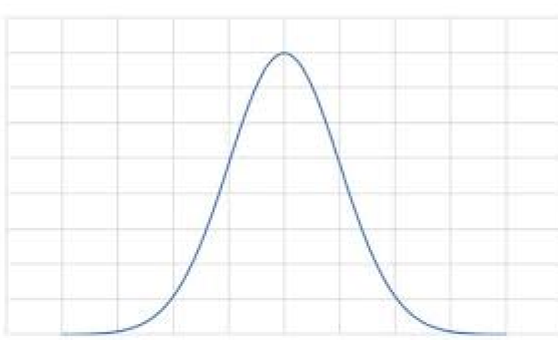
Goal: To be able to read and interpret a normal distribution.

A normal distribution is a special type of distribution. We can use histograms to demonstrate normal distributions. We can also use a smooth curve.

In a normal distribution, the mean, median and mode all have the same value.

Examples:





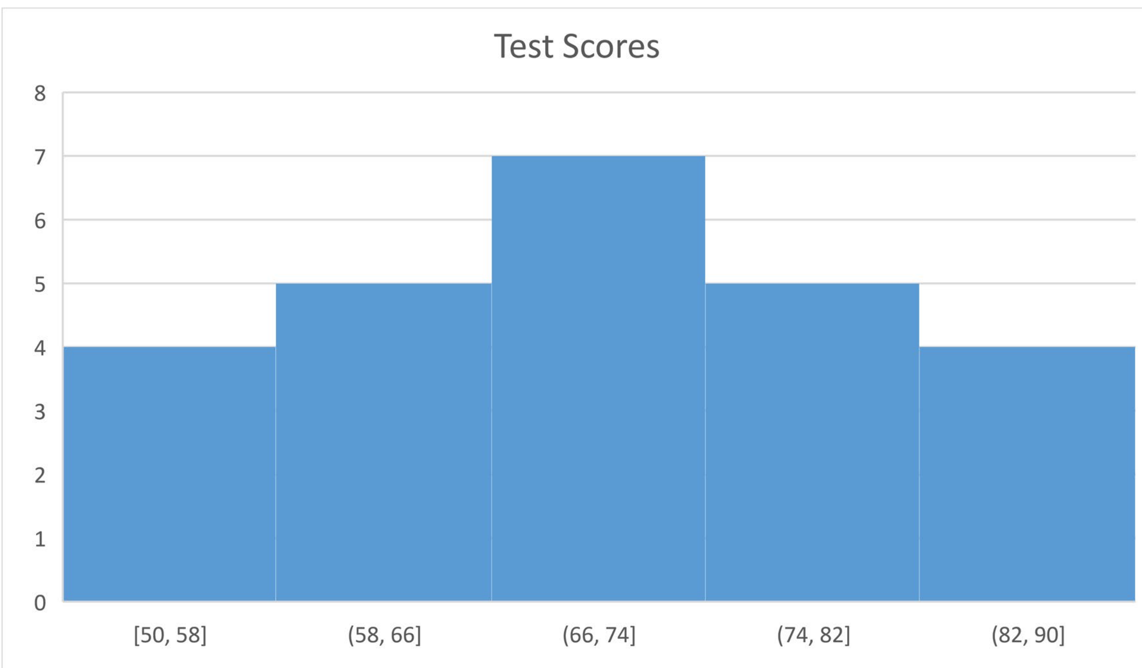
Properties of a Normal Distribution:

1) The graph is called a “normal curve”.
2) The curve is bell-shaped and symmetric about the mean.
3) The mean, median and mode all have the same value. This value is at the center of the curve.

Empirical Rule:
In any normal distribution:

1) Approximately 68% of all the data lies within one standard deviation of the mean on both sides.
2) Approximately 95% of the data lies within two standard deviation of the mean on both sides.
3) Approximately 95% of the data lies within two standard deviation of the mean on both sides.

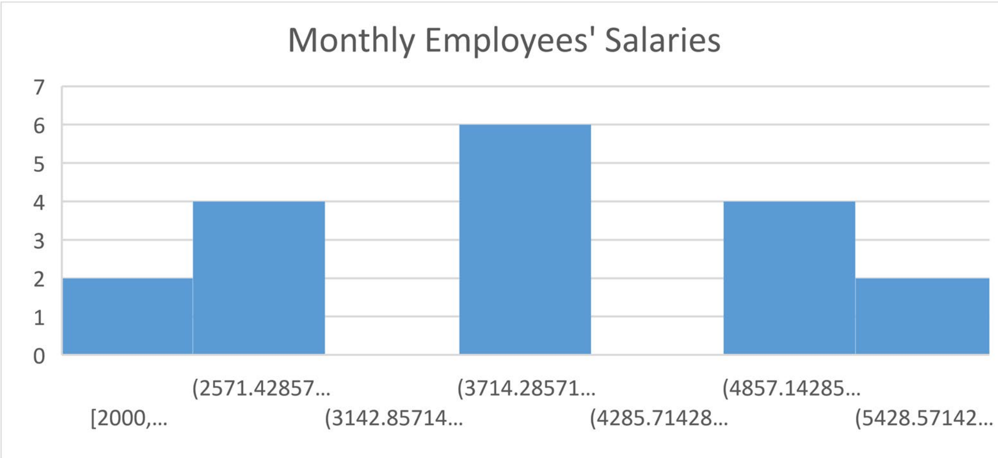
Example:



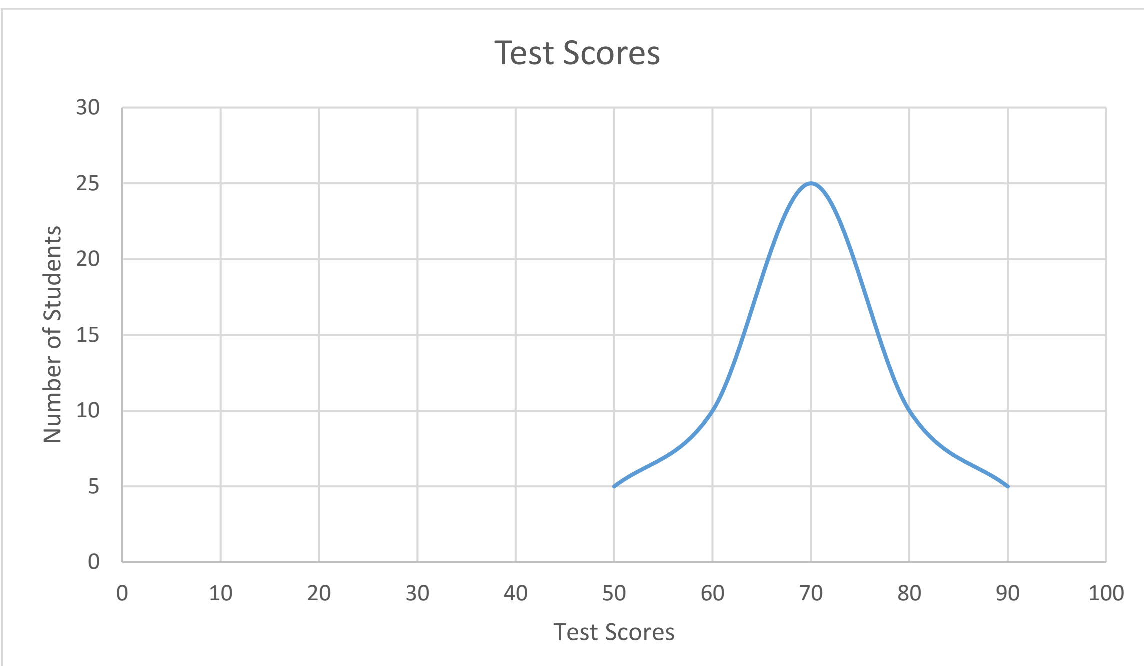
The histogram above is a normal distribution:

1) What is the mean? Since the middle bar goes from 66 to 74, we see the middle value is 70.
2) How many students had a score between 58 and 66? 4.
3) Which other scores had the same amount of students as number 2)? 74-82.

Exercise Set 4:



1) What is the mean employee salary?
2) How many employees had a monthly salary between $2,500 and $3500/mo?
3) What was the other salary range for 4 other employees not in that range?
4) How many employees were in the mean salary?



The bell-curve above (end of previous page) is another normal distribution.

1) What is the mean?
2) How many students received a score that was the mean?
3) How many total students were there?
4) What was the highest test score?
5) What was the lowest test score?
6) What was the range of test scores? (Hint: Highest minus lowest).
7) How many students got a 60%?
8) How many students got a 90?

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8^{th} ed.
    - Chapter 13.4, 13.7
- Mathematical Excursion...
    Chapter 13.4

Required Topic 7b: Construct and analyze statistical graphs: Labeling the normal distribution bell curve given the mean & standard deviation.

MAT-175 Required Topic 7b is intended to support the following MAT-120 topics:

1. Probability and Statistics: Table and chart analysis
2. Probability and Statistics: Normal distribution

Khan Academy Support https://www.khanacademy.org/math/statistics-probability/modeling-distributions-of-data/more-on-normal-distributions/v/introduction-to-the-normal-distribution

Goal: To demonstrate competence in labeling the normal distribution given the mean & standard deviation. (Note: In the previous section, we learned how to read and interpret a normal distribution graph).

In the previous section, we discovered:

Properties of a Normal Distribution:

1) The graph is called a "normal curve".
2) The curve is bell-shaped and symmetric about the mean.
3) The mean, median and mode all have the same value. This value is at the center of the curve.

Empirical Rule:
In any normal distribution:

4) Approximately 68% of all the data lies within one standard deviation of the mean on both sides.
5) Approximately 95% of the data lies within two standard deviation of the mean on both sides.
6) Approximately 95% of the data lies within two standard deviation of the mean on both sides.

Recall, we learned about the standard deviation in section 2e.

This is what we covered:

The formula is: $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$. Another symbol for the standard deviation is σ.

Let us go through the formula: s is the standard deviation (we will go through this in more detail in another section). x is each number in your set of data. $\bar{x}$ is the mean you must calculate. $\sum$ means sum (or to add). n is the number of entries in your set of data.

Example: Find the mean and standard deviation for the following set of data: 1,2,2,3,4,6:

First we find the mean: $\frac{1+2+2+3+4+6}{6} = 3.$

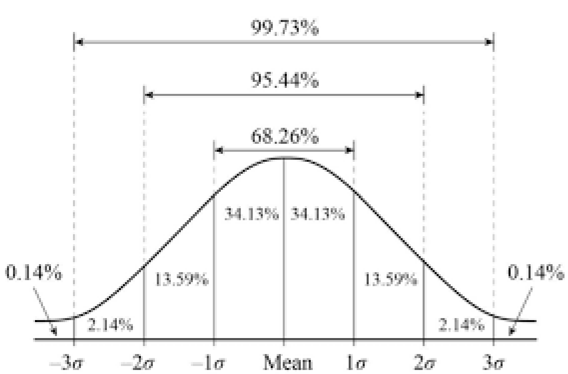
Next we can substitute everything into our formula. $s = \sigma = \sqrt{\frac{(1-3)^2+(2-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(6-3)^2}{6.-1}}$. Next, we will simplify the numerator under the radical. We can do this in our calculator or by inspection: $s = \sqrt{\frac{(-2)^2+(-1)^2+(-1)^2+0^2+1^2+3^2}{5}}$. We continue to simplify: $s = \sqrt{\frac{4+1+1+1+9}{5}} = \sqrt{\frac{16}{5}} =$ Now we put this into our scientific calculator: We divide $\frac{16}{5}$ to get: 3.2. We then hit the square root button to get: ≈ 1.789

Let us now explore the concept of Standard Deviation a little further:

Standard Deviation: Is another measure of "dispersion" or "spread of data", as is the range. It gives us information on whether the data is spread farther away from the mean, or if it is clustered close to the mean. The larger the spread, the higher numerical value the standard deviation will have.

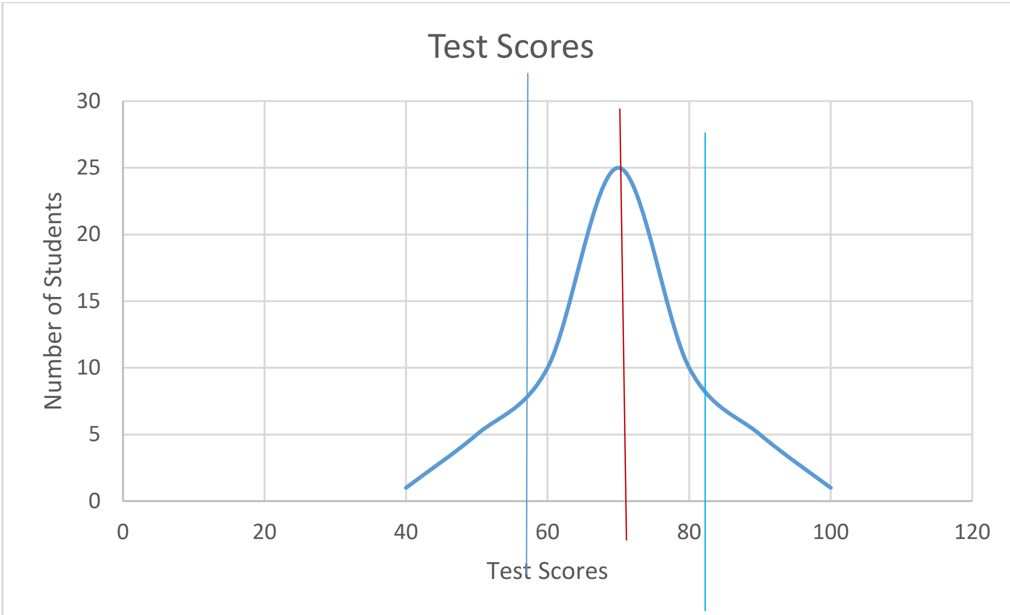
Example: Let's say there were 2 math classes, both of which had a mean of 70. Let's say the first class had many scores in both the 20's and the 90's. This class would have a large standard deviation. Let's say in the second class most of the scores were in the 70's with a few in the 60's and 80's. The second class would have a small standard deviation.

Here is a properly labeled normal distribution bell curve:



Note: The mean is the highest point on the curve. The standard deviation is on either side. We can label $1\sigma, 2\sigma, etc.$

Example:



We have labeled the mean in red.

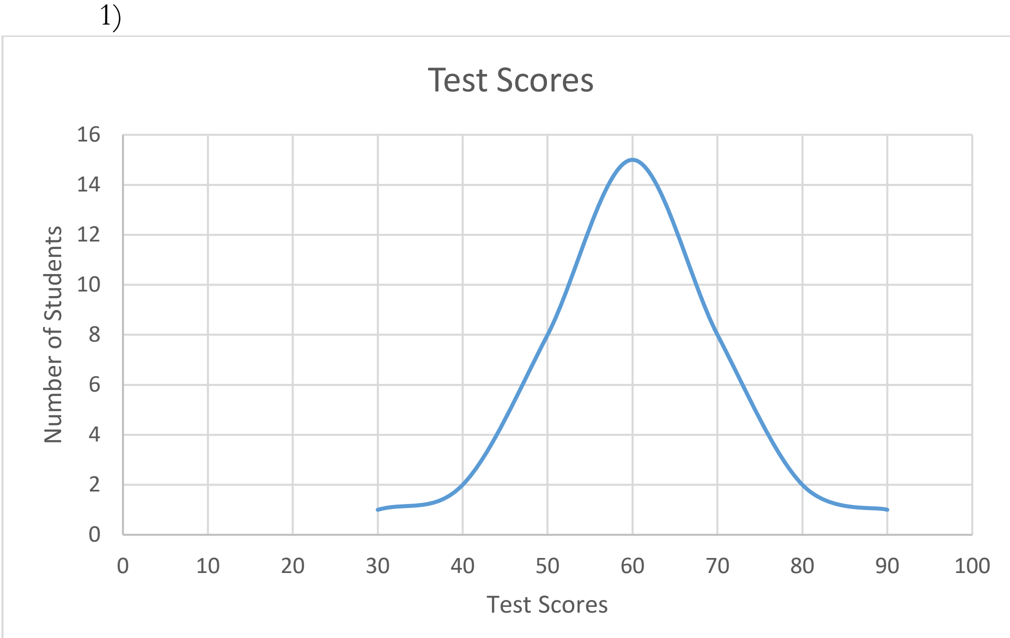
$s = \sigma = 11.7$

We have labeled the first standard deviation is green.

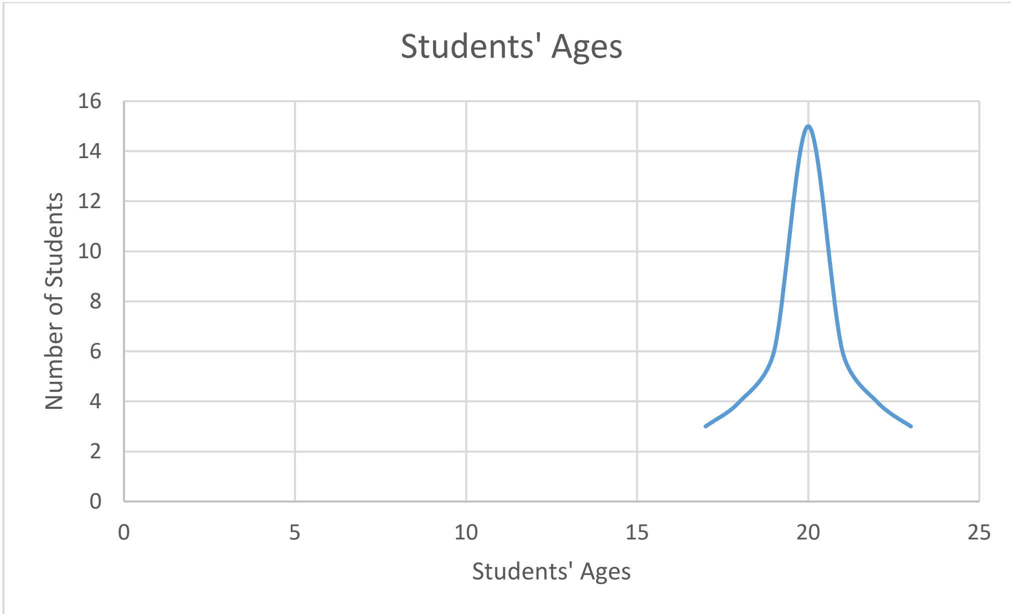
Exercise Set 1:

Label the mean and the first standard deviation for the following graphs:

1)



The standard deviation is 11.6 for graph #1).

2) 

The standard deviation is 1.55 for graph #2).

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 13.7
- Mathematical Excursion...
  Chapter 13.4

Required Topic 7c: Construct and analyze statistical graphs: Labeling the area of the bell curve of interest.

MAT-175 Required Topic 7c is intended to support the following MAT-120 topics:

1. Probability and Statistics: Table and chart analysis
2. Probability and Statistics: Normal distribution

Khan Academy Support: https://www.khanacademy.org/math/ap-statistics/density-curves-normal-distribution-ap/normal-distributions-calculations/v/z-table-for-proportion-between-values

Goal: To be comfortable labeling different areas under the bell curve for given z-scores.

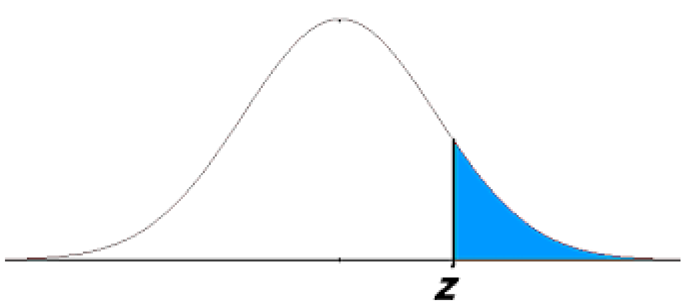
Z-Scores: (Also known as standard scores) will allow us to determine how far a given score is from the mean. It will be it terms of the standard deviation. (We covered this briefly in 4b).

Z-Score Formula: $z = \frac{Value\ of\ piece\ of\ data - mean}{standard\ deviation} = \frac{x-\mu}{\sigma}$, where x is the value of the piece of data of interest, μ is the mean, and σ is the standard deviation.

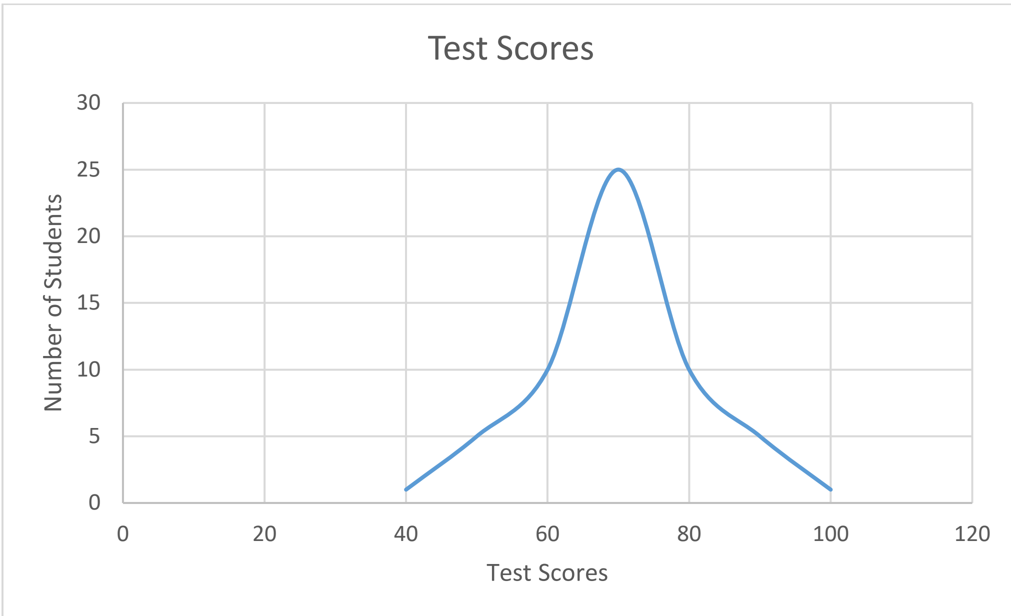
Data above the mean will have positive scores while data below the mean will have negative values.

We can easily calculate the z-score for a piece of data using the above formula. We will then have to use a table to find the area under the curve for a given z-score. We will not duplicate the table in this section. We will instead, give you the areas from the table in order to complete the problems. In your MAT 120 course, you will learn more about how to use the table.

Example:

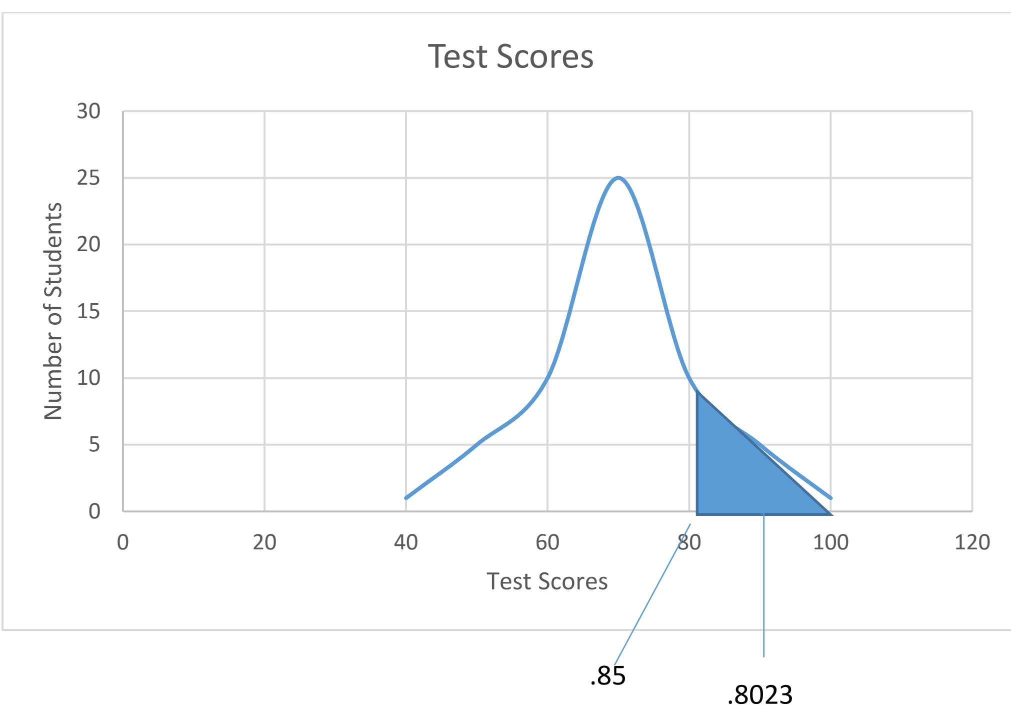


Example: Let's go back to an example we did in the previous section.

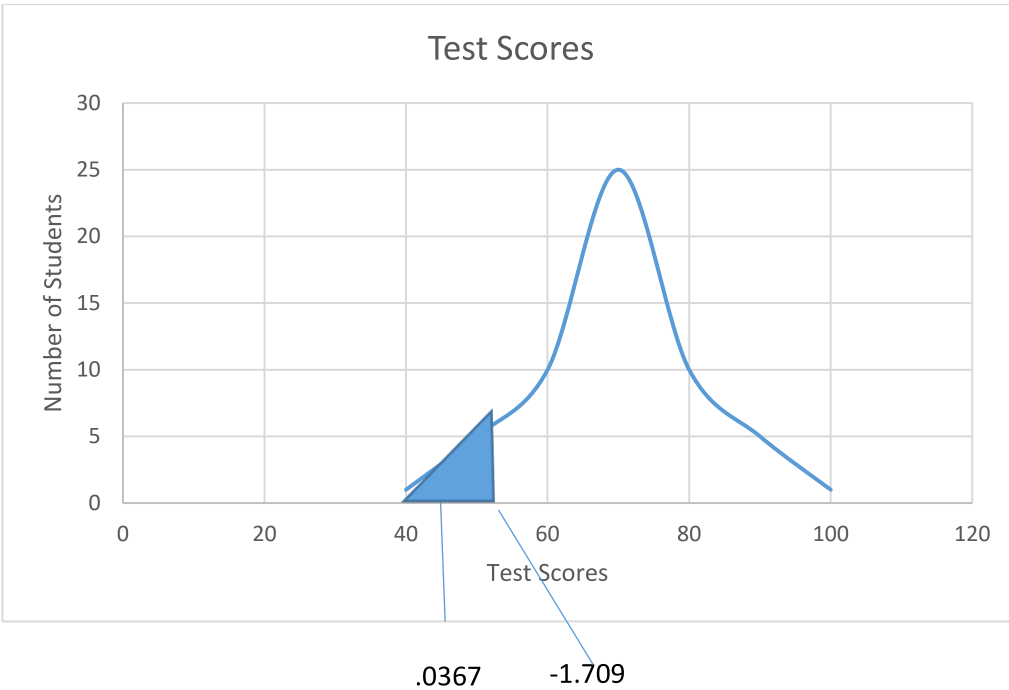


Recall $\sigma = 11.7$. Also recall $\mu = 70$, (which we can also quickly observe by looking at the graph). Let us determine the z-scores for a) 80, b) 50: Then we will find the area, shade and label.

a) $z = \frac{x-\mu}{\sigma} = \frac{80-70}{11.7} = .85$. Let us find the area to the right of z=.85. Now we have to use a table. (**Note: The tables we have give us the areas to the LEFT of the z-value.** The area is .8023. This is the area to the left of .85. To get the area to the right, we subtract .85 from 1 to get .15. (This is because the area under the normal curve is 1). Let us appropriately label the graph.

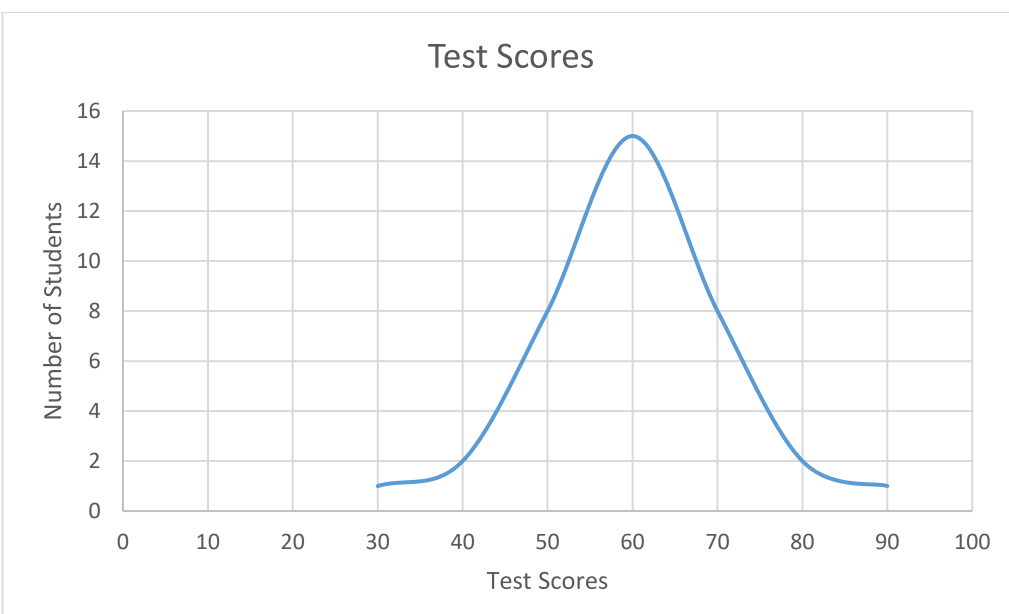


b) $z = \frac{x-\mu}{\sigma} = \frac{50-70}{11.7} = -1.709$. Now we have to use a table. The z-score is negative, because 50 is to the left of the mean. Using the table, the area is .0367. Let us appropriately label the graph.



Exercise Set 1:

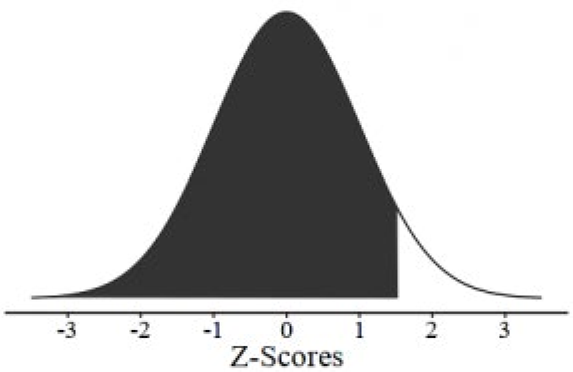
1)



Recall this graph from the previous section in Exercise Set 1. You have already labeled the mean and one standard deviation in that exercise. We recall the mean, $\mu = 60$, and the standard deviation, $\sigma = 11.6$.

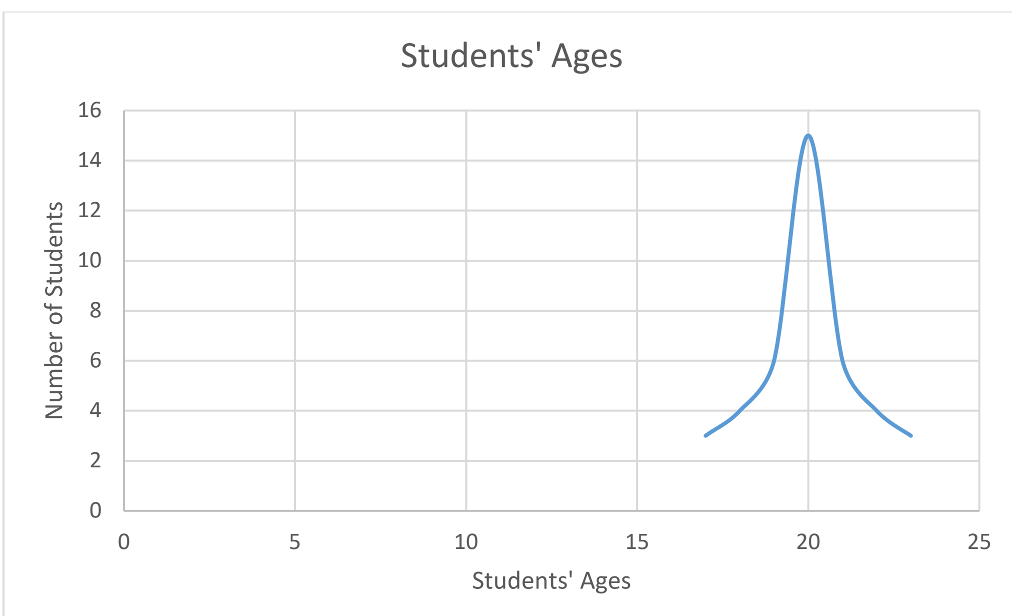
a. We will now find the z-score for 75. $z = \frac{x-\mu}{\sigma} = \frac{75-60}{11.6} = 1.29$. Using the table, we find the area to be, .9015. This is the area to the left of z = 1.29 (or the x-value of 75). Now shade and label the area:

Hint: We didn't do an example that this will look like. Here is an example of a z-score to the right of the median, with the area to the left:



b. We will now find the z-score for 40. $z = \frac{x-\mu}{\sigma} = \frac{40-60}{11.6} = -1.72$. (It is negative, because 40 is to the left of the mean. Using the table, we find the area to be, .0427. This is the area to the left of z =- 172. Now shade and label the area.

2)



This is the other graph from the previous section from Exercise Set 1.
You have already labeled the mean and one standard deviation in that exercise. We recall the mean, $\mu = 20$, and the standard deviation, $\sigma = 1.55$.

a. We will now find the z-score for 22. $z = \frac{x-\mu}{\sigma} = \frac{22-20}{1.55} = 1.29.$ Using the table, we find the area to be, .9015. This is the area to the left of z = 1.29 (This is pure coincidence that the z-score is the same as our example). Let us find the area the right. 1 - .9015 = .0985. Now shade and label the area:

b. We will now find the z-score for 17. $z = \frac{x-\mu}{\sigma} = \frac{17-20}{1.55} = -1.94.$ Using the table, we find the area to be, .0262. This is the area to the left of z = -1.94. Shade and label the area to the left of z = -1.94.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 13.7
- Mathematical Excursion...
    Chapter 13.4

Recommended Topic 1a: Mathematical organization, and critical thinking skills: Apply exponential modeling to real-world applications other than compound interest.

MAT-175 Recommended Topic 1a is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Application of basic exponential models
2. Apply Math Modeling to Real World Applications: Construction of basic exponential models

Goal: To be comfortable exploring a variety of basic exponential models relating to exponential growth and decay in order to construct and apply the models in MAT 120.

We covered some of this in Section 6a. We will go a bit further in this section. Let us review what we did there:

REVIEW:

Exponential Functions: To recap:

The exponential equation: $y = a^x$, where $a > 0, a \neq 1$, satisfies the vertical line test (covered in the previous section). Therefore the exponential equation: $y = a^x$ is a function. We could use the notation, $f(x) = a^x$, but we will save that notation for your MAT 120 class.

Let us briefly discuss why $a > 0, a \neq 1$. If $a = 0$, or if $a = 1$, we would not have an exponential function, but a linear one, as 0 times itself over and over is 0, and likewise for 1. If $a < 0$, we would get a lot of imaginary numbers, which is not what we want.

An exponential function always goes through the point (0,1), will always be constantly increasing or constantly decreasing, and will have a horizontal asymptote at the x-axis, or $y = 0$. The asymptote, in this case, is a line in which the function will get closer and closer, but will not cross or touch. In this case, it will only approach the asymptote in one direction, depending whether the function is increasing or decreasing.

We observed that when $a > 1$, the function was increasing everywhere. It was getting closer and closer to the asymptote $y = 0$ on the left side of the graph. The further left we went, the closer it got. On the right side, it got larger and larger the farther you traveled. If you think about this, it makes sense. If you have a number >1, and you keep multiplying it by itself over and over, it will grow rapidly. When x is negative, it gives the reciprocal, so that when multiplying it by itself over and over, it decreases rapidly until it gets very close to zero. We observe it can never by 0 or negative. When $0 < a < 1$, we observe the opposite is true. We also recall that $a > 0, a \neq 1$.

Basic Formula:

Let us begin by breaking down the basic exponential formula: $y = a^x$. We are already familiar with y as the output, or **dependent**, variable. a is the base. The base is the number you will multiply over and over x times. x is the input, or **independent**, variable. We choose values for x, and find y based on our choice for x. We note that a, the base, is a constant. A constant is a number, a value that does not change. a could be 2 for example. Or it could be $\frac{1}{3}$. a can be any real number except a must be >0, and recall that $a \neq 1$. It can be any number other than the restrictions listed. x can be any real number. We call these values the **Domain.** These are all the allowable input values. y will always be >0, or $(0, \infty)$. We call these values the **Range.** These are all the possible output values we could obtain.

Review: Let us quickly review some basic rules of exponents:

1) $a^m \cdot a^n = a^{m+n}$
2) $\frac{a^m}{a^n} = a^{m-n}$
3) $(a^m)^n = a^{m \cdot n}$
4) $a^{-n} = \frac{1}{a^n}$
5) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
6) $(ab)^n = a^n \cdot b^n$
7) $\left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

Then we did some exponential growth and decay:

Exponential Growth and Decay:

The formula for exponential growth and decay is basically the same one as the continuously compounded interest one: It is:

$$P = P_0 e^{rt}$$

Where P is the final population, P_0 is the initial population, r is the growth or decay rate, and t is time. In this case t may or may not be in years. It depends on the context of the problem, and what you are measuring. r will be positive in a population growth problem, and negative in a decay problem.

Example:

Math For Liberal Arts class doubles every 10 days, because all students know it is the best thing happening on Earth! Find the growth rate, r.

To do this problem we observe that P is 2 and P_0 is 1. Why? It doubles, which is always 2 to 1. No matter how many we start with, we end up with twice as much. t is 10 days.

We insert this info into $P = P_0 e^{rt} \rightarrow \quad 2 = e^{r \cdot 10}$. To solve for r, we will require the understanding of logarithms. This is beyond this course. I wanted to present it, just to show how it gets used for future reference. These problems recur often in : Biology, chemistry, mathematics, physics, etc. You will likely encounter them frequently.

Exercises from Section 6a:

For the following set of exercises, identify P, P_0, t:

1) A certain population triples every 2 years.
2) A substance has a ½ life of 3 hours. (Hint: whatever you start with, your final population is ½).

Example:

What is the final population, if we start with 100, the growth rate is 2%, and the time is 5 hours?

$$P = P_0 e^{rt} \rightarrow \quad P = 100e^{.02 \cdot 5}$$

First we multiply .02 times 5 to get .1. Next, we raise e to the power of .1 to get 1.105. Finally we multiply 1.105 by 100 to get 110.5 This is our final population.

Exponential Growth and Decay:

The formula for exponential growth and decay is basically the same one as the continuously compounded interest one: It is:

$$P = P_0 e^{rt}$$

Where P is the final population, P_0 is the initial population, r is the growth or decay rate, and t is time. In this case t may or may not be in years. It depends on the context of the problem, and what you are measuring. r will be positive in a population growth problem, and negative in a decay problem.

Example:

Math For Liberal Arts class doubles every 10 days, because all students know it is the best thing happening on Earth! Find the growth rate, r.

To do this problem we observe that P is 2 and P_0 is 1. Why? It doubles, which is always 2 to 1. No matter how many we start with, we end up with twice as much. t is 10 days.

We insert this info into $P = P_0 e^{rt} \rightarrow \quad 2 = e^{r \cdot 10}$. To solve for r, we will require the understanding of logarithms. This is beyond this course. I wanted to present it, just to show how it gets used for future reference. These problems recur often in : Biology, chemistry, mathematics, physics, etc. You will likely encounter them frequently.

Exercises from Section 6a:

For the following set of exercises, identify P, P_0, t:

1) A certain population triples every 2 years.
2) A substance has a ½ life of 3 hours. (Hint: whatever you start with, your final population is ½).

Example:

What is the final population, if we start with 100, the growth rate is 2%, and the time is 5 hours?

$$P = P_0 e^{rt} \rightarrow \quad P = 100e^{.02 \cdot 5}$$

First we multiply .02 times 5 to get .1. Next, we raise e to the power of .1 to get 1.105. Finally we multiply 1.105 by 100 to get 110.5 This is our final population.

Let us now do some more examples of exponential growth and decay.

Example 1:

A new type of mammal never before seen has a population with a growth rate of 15%. If the initial population was 10, what will the population be after 10 years?

Let us first identify, P_0, r, t: $P_0 = 10,\ r = 15\% = .15,\ t = 10.$

Next, we substitute the values into the formula: $P = P_0 e^{rt} \rightarrow \quad P = 10e^{.15(10)} = 44.8$ or about 45 new mammals.

Example 2:

A newly discovered bacteria has a population that doubles every 5 seconds. If we start out with 100 bacteria, how many will we have after an hour?

This problem requires some more advanced mathematics and the use of logarithms (which are beyond the scope of this class, and not a topic in MAT 120). Because of this, I will do that for you, and give you the final result, so you can continue. I must find our growth rate, r. We start with the formula: $2 = e^{r \cdot 5}$. We got this formula, because the final population is twice the initial after 5 seconds. (Note: There is actually a 1 multiplied by e, but we normally omit it as it is implied). I must now solve for r. (After some math, we find r to be equal to .139). (This is in terms of 5 seconds).

Next we convert our hour to seconds. (All units must match).
1 hour = 3600 seconds.

Next, we substitute everything into our formula: $P = P_0 e^{rt} \rightarrow \quad P = 100e^{.139(3600)} = 2.1 \times 10^{219}$. Wow! That's a lot of bacteria!

Example 3:

A certain type of newly discovered radioactive substance has a half-life of 6 days. How much will be left after 1 month if we start with 2000 grams?

I must find our growth rate, r. We start with the formula: $1 = 2e^{r6} \rightarrow \frac{1}{2} = e^{6r}$. We got this formula, because the final population is 1/2 the initial population after 6 days. I must now solve for r. (After some math, we find r to be equal to -0.116). Notice that our growth rate is negative this time. This is because it is a **decay** problem rather than a growth problem. This is what we expect for these types of problems. r, positive for growth problems, and negative for decay problems. Decay problems are frequently half-life, or any type of radioactive decay.

Next, we convert months to days. We will use 30 days as a good approximation for a month.

$$P = P_0 e^{rt} \rightarrow \quad P = 2000e^{-0.116(30)} = 62.5\ g.$$

Exercise Set 1:

1) Joyce's dogs have a population with a growth rate of 5%. If the initial population was 4, what will the population be after 10 years?

2) Sally is raising some rabbits. If the rabbits have a growth rate of 12%, and the initial population is 20, what will the population be after 4 years?

3) Jonathan has some frogs. If the frogs have a growth rate of 10%, and the initial population is 200, what will the population be after 5 months?

4) A newly discovered radioactive substance has a decay rate of -.2. If the initial population is 5000 g, how much will be left after 2 weeks? (Note: you can use whatever units you like. Just make sure you use the same units in your final answer).

5) A newly discovered bacteria has a population that doubles every 2 hours. If we start out with 1500 bacteria, how many will we have after 10 hours? You will need the growth rate, r, which is $.347$.

6) A certain type of newly discovered radioactive substance has a half-life of 3 days. How much will be left after 2 months if we start with 25,000 grams? The decay rate, r is $-.23$.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 6.3
- Mathematical Excursion...

  Chapter 10.5

Recommended Topic 2a: Appropriate use of technology: Quadratic regression

MAT-175 Recommended Topic 2a is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Graph analysis in the context of an application

Goal: To be comfortable performing quadratic regression using the technology of your choice.

Tutorials:

Ti-84: https://www.youtube.com/watch?v=74bdyEF6Ugw

Ti-89: https://www.youtube.com/watch?v=pJ6eQPBy21Y

Desmos: https://www.youtube.com/watch?v=8i741SSGxPQ

Excel: https://www.youtube.com/watch?v=kFasmbrDG5I

Recommended Topic 2b: Appropriate use of technology: Exponential regression

MAT-175 Recommended Topic 2b is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Application of basic exponential models
2. Apply Math Modeling to Real World Applications: Construction of basic exponential models

Tutorials:

Ti-84: https://www.youtube.com/watch?v=TkMQ5n6vWGg

Ti-89: https://www.youtube.com/watch?v=cD4e682Js0o

Desmos: https://www.youtube.com/watch?v=3y_GeG88wgw

Excel: https://www.youtube.com/watch?v=aw-GluLZIWA

Recommended Topic 3a: Understanding of mathematical notation: Determining rows/columns for truth tables.

MAT-175 Recommended Topic 3a is intended to support the following MAT-120 topics:

1. Formal Logic: Tests for validity: Truth Tables

Goal: To be more comfortable with truth tables in general, and to be able to comfortable determine how many columns are needed in any given truth table.

YouTube Support: https://www.youtube.com/watch?v=fbuRxa9MNPM

Truth Tables:

A truth table is an array of information that allows us to determine if a compound statement is true or false. It will give us the answers for all possible scenarios.

We learned about logic statements in Section 4b.

We have found that many MAT 120 students struggle with how many rows and columns we need for a truth table. This is especially true for the number of columns.

Let us review basic truth tables:

First, let us write down the basic truth tables for p, q; and for p, q, r. These are the tables that give all possible combinations of each simple statement:

Basic Table for p, q:

p	q
T	T
T	F
F	T
F	F

We observe this is every combination possible for T,F for two simple statements:

Basic Table for p, q, r:

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

We observe this is every combination possible for T,F for three simple statements:

Note: When filling out a truth table, we copy these down first! These tables are given, and we can simply copy each column as it occurs in any truth table.

Negation: Recall negation is a "not" statement we write as ~.

Example: If we have ~p, we would simply negate (or "flip") the column:

p
T
T
T
T
F
F
F
F

Becomes

~p
F
F
F
F
T
T
T
T

Notice that each T becomes F, and each F becomes T. We can negate any column using this method.

Example 1: Let's take a compound statement from an exercise in Section 4b: $p \wedge (q \vee r)$. You were to list the Order of Operations.

I will create a truth table for this statement using the p, q, r table from above, and the order of operations. I will do it one step at a time. **(In this section, I will not have you construct truth tables; but merely have you determine how many rows and columns you need).** I will construct a few, so you can see how to determine the rows and columns needed.

$\boldsymbol{p}$	$\wedge$	$(\boldsymbol{q}$	$\vee$	$\boldsymbol{r})$
T		T		T
T		T		F
T		F		T
T		F		F
F		T		T
F		T		F
F		F		T
F		F		F

So far, all I have done is copy the columns for p, q, r. Note that they are the same as the given table.

Next, we will complete the table by using the proper Order of Operations we learned in Section 4b.

Recall, that next, we complete the column for "or", $\vee$. "Or" is true, if one or both are true. Example: If I say I am holding a blue pen "or" a red pen; then it is true if I am holding one, or the other, or both. The only time that is a false statement, is if I am not holding a blue or a red pen.

Let us now complete that column:

$\boldsymbol{p}$	$\wedge$	$(\boldsymbol{q}$	$\vee$	$\boldsymbol{r})$
T		T	T	T
T		T	T	F
T		F	T	T
T		F	F	F
F		T	T	T
F		T	T	F
F		F	T	T
F		F	F	F

Finally, we will complete the "and", $\wedge$, column.

Before filling out this column, we need to determine 2 things:

1) What 2 columns are we are comparing.
2) When an "And" statement is true or false.

1) We always compare the connectives, if there is one. In this example, the connective we have filled out is "or". This is the column we just completed. If there is a connective on the other side, we compare the two connectives. In this case, there is only one thing on the other side, and it is the column, $\boldsymbol{p}$. These will be the two columns we compare to complete the "and" column.
2) Next, we complete the column for "and", $\wedge$. "And" is only true, if both are true. Example: If I say I am holding a blue pen "and" a red pen; then it is only true if I am holding both a blue and a red pen. All other cases are false.

We finish the table:

$\boldsymbol{p}$	$\wedge$	$(\boldsymbol{q}$	$\vee$	$\boldsymbol{r})$
T	T	T	T	T
T	T	T	T	F
T	T	F	T	T
T	F	F	F	F
F	F	T	T	T
F	F	T	T	F
F	F	F	T	T
F	F	F	F	F

Normally we circle the final column, to show this is our final answer. Instead, I will reproduce the column in red:

$\boldsymbol{p}$	$\wedge$	$(\boldsymbol{q}$	$\vee$	$\boldsymbol{r})$
T	T	T	T	T
T	T	T	T	F
T	T	F	T	T
T	F	F	F	F
F	F	T	T	T
F	F	T	T	F
F	F	F	T	T
F	F	F	F	F

We need to observe that each operation gets its own column. Each statement has a column, and each connective also has a column.

In the interest of brevity, I will not construct a variety of truth tables; and I will not carefully go through the other tables for conditional and bi-conditional ("if, then", and "if and only if)". Instead I will construct a variety of logic statements, and we will determine how many rows and columns each one would have. We will then create the columns and rows without finishing the tables.

Example 2:

Let us construct a truth table for the following statement: (Note: we will not complete the table).

$$(p \vee q) \wedge (p \vee \sim r).$$

$(p$	$\vee$	$q)$	$\wedge$	$(p$	$\vee$	$\sim r)$

We will not fill this out any further. Let us discuss why we have these rows/columns. For any table with 3 simple statements: $p, q, r,$ we will have 9 rows, the first row for the statement, and 8 rows below it. (We are focusing on p, q, r rather than $p, q,$ because these are the tables that give MAT 120 students the most difficulties.)

We further note that each statement and connective gets its own column. **Note: We have $\sim r$ as its own column. We could have done 2 columns, one for r, and one for ~, if we had wanted to. (It seems redundant to me, so I did it in one column.). There are cases where ~ will need its own column.** Let's explore an example where it will:

$$q \wedge \sim(p \vee r)$$

q	$\wedge$	$\sim$	$(p$	$\vee$	$r)$

In this table, ~ gets its own column, because it is negating the entire compound statement: $p \vee r$, rather than a simple statement such as p, q or r .

Exercise Set 1:

For the following set of exercises determine the following:

a) Determine the number of rows for the impending truth table:
b) Determine the number of columns for the impending truth table:
c) Construct the truth table with the first row filled out (this is the statement given in the problem):
d) Fill out the simple statements for p, q, r or their negations.
e) You do not need to complete the truth table.

1) $\sim p \wedge (q \vee r)$
2) $r \vee \sim (p \wedge q)$
3) $q \rightarrow (p \vee r)$
4) $\sim r \leftrightarrow \sim (p \vee q)$
5) $q \wedge \sim (p \leftrightarrow r)$
6) $p \rightarrow \sim (q \vee r)$
7) $q \wedge \sim (p \wedge r)$

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 3.2, 3.3.
- Mathematical Excursion...
  Chapter 3.2, 3.3

Recommended Topic 3b: Understanding of mathematical notation: Understanding the difference between truth tables and truth values.

MAT-175 Recommended Topic 3b is intended to support the following MAT-120 topics:

1. Formal Logic: Tests for validity: Truth Tables
2. Formal Logic: Introduction to Conjunction, Disjunction, & Negation

Goal: To demonstrate a complete understanding of the difference between truth table and truth values.

Many MAT 120 students confuse these two very different topics.

The most frequent mistake I see is for a student to give me a truth table when I have asked for a **Truth Value.**

In the previous section, we have introduced the idea behind truth tables. We have concentrated our discussion on the rows and columns. (With emphasis on how to determine the columns).

In this section, we will concentrate our discussion on Truth Values, and how it is different from a Truth Table.

The Main Difference: A Truth Value is a "SINGLE VALUE.", either True or False. Whereas, a Truth Table, is an array of values. For a Truth Value, the statements p, q, r must be assigned values of True or False. A Truth Table will have a column of possible values for each statement p, q, r.

Example 1:

Let us determine the truth value for the following statement if p is true, q is true, and r is false:

$p \wedge (q \vee r)$. (Note: We constructed a truth table for this statement above. That is not what we will do here).

Let us instead, place values for each statement: $p \wedge (q \vee r)$ *becomes* $T \wedge (T \vee F)$. Next we use the Order of Operations to conclude the validity. We perform the operation inside the parentheses: $T \vee F = T$. We then compare the T on the left to the T on the right we get $T \wedge T$ is T. True is the answer to this statement.

Example 2:

Let us determine the truth value for the following statement if p is false, q is false, and r is true:

$(p \vee q) \wedge (p \vee {\sim} r)$ *becomes* $(F \vee F) \wedge (F \vee F)$. (Note: if r is true, then ${\sim} r$ is false. That's where the last false came from. Everything is False. The Statement is False. (When everything is false, both "and" and "or" statements are false. Not true for conditional or bi-conditional statements: if then, or if and only if).

Exercise Set 1:

For the following set of exercises determine the following: (Note: Some of these statements are the same statements from Exercise Set 1 from the last section). We want to further show how different the answers are from that section. (Note: I have omitted conditional and bi-conditional statements).

Assume for all statements that p is true, r is false, and q is true.

a) Copy the statement.
b) Write T or F below each element of the simple statement, p, q, r.
c) Complete the rest of the compound statement by writing T or F below the connectives using the Order of Operations.
d) Determine the truth value.

1) $\sim p \wedge (q \vee r)$
2) $r \vee \sim (p \wedge q)$
3) $q \vee (p \vee r)$
4) $\sim r \wedge \sim (p \vee q)$
5) $q \wedge \sim (p \wedge r)$

Exercise Set 2:

For the following exercise, determine if you will have an array of values, or one final value:

$$p \wedge (q \vee r)$$

1) Find a truth table for the above statement:
2) Find a truth value for the above statement if p, q, r are all true.

Note: You do not have to construct a truth table, or find a truth value.

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
    - Chapter 3.2, 3.3.
- Mathematical Excursion...

  Chapter 3.2, 3.3

Recommended Topic 4a: Techniques for graphing functions without the use of a calculator: Graphing quadratic functions.

MAT-175 Recommended Topic 4a is intended to support the following MAT-120 topics:

1. Math Modeling/Real World Applications: Graph analysis in the context of an application

Khan Academy Support https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:standard-form-quadratic/v/graphing-a-parabola-using-roots-and-vertex

Goal: To be able to graph a Quadratic Function without aa calculator, by finding the vertex, knowing the shape, and determining the x and y intercepts.

Point-Plotting Method: We learned the general point-plotting method in Section 5a. You can use this method to graph any quadratic function, however, it is not the most efficient method. We will leave this method as an exercise for the student, if interested.

General Form for a Quadratic Function: $\boldsymbol{y = ax^2 + bx + c}$, where ax^2 is the leading term, and a is called the leading coefficient.

General Method:

Properties of graphs of Quadratic Functions:

1) We know the shape of a quadratic function is a parabola (cup-shaped curve).
2) It will open "up", or it will open "down". It opens upward if a is positive, and downward if a is negative.
3) It has a vertex: The vertex is the lowest point of the parabola if it opens up, and the highest point if it opens down. Some call it a "turning point".
4) It will have a y-intercept. It may or may not have x-intercepts.

Vertex Formula: $x = -\frac{b}{2a}$, $y = f\left(-\frac{b}{2a}\right)$. To find y, you substitute, $x = -\frac{b}{2a}$, into x to find y. To use the vertex formula, you use the values from: $\boldsymbol{ax^2 + bx + c}$. (Note: the x-value from the vertex formula is also known as the axis of symmetry). (Note: There is another method to find the vertex, called Completing the Square to put the function in standard form. We will not explore that here).

Example 1: Find the vertex for the following quadratic function: $y = 2x^2 - 3x + 1$. We first denote $a = 2,\ b = -3,\ c = 1$. We now substitute these values into the formula $x = -\frac{b}{2a}$ to find the x-value of the quadratic formula: $x = -\frac{b}{2a} = -\left(-\frac{3}{2\cdot 2}\right) = \frac{3}{4}$.

Now we substitute this into x to find y: $2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1 = \frac{9}{8} - \frac{9}{4} + 1 = \frac{9}{8} - \frac{18}{8} + \frac{8}{8} = -\frac{1}{8}$. Our vertex is: $\left(\frac{3}{4}, -\frac{1}{8}\right)$.

Exercise Set 1:

For the following set of exercises, find the vertex for the following quadratic functions, as in the above example:

1) $y = x^2 + 4x - 9$
2) $y = x^2 - 2x + 7$
3) $y = 2x^2 + 8x - 3$
4) $y = 4x^2 + 16x - 2$
5) $y = 2x^2 - 5x$

Steps for Graphing a Quadratic Function:

1) Determine if the parabola opens upward or downward.
2) Find the vertex: First find the x-value using the Vertex Formula, then find the y-value.
3) Find the y-intercept.
4) Find any x-intercept(s). (They may or may not exist. There could be one or two).
5) Sketch the graph.

Example 2: Let $y = x^2 - 6x + 5$

Let us graph this function using steps 1) – 6) above:

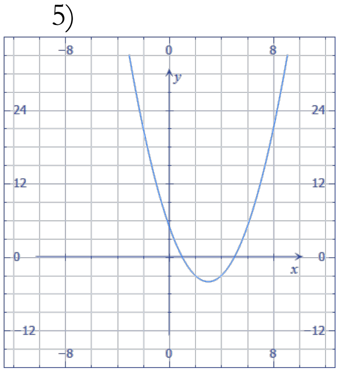
1) $a = 1$, which is positive. Therefore our parabola opens upward.

2) $x = -\frac{b}{2a} = \frac{6}{2 \cdot 1} = 3$. $y = 3^2 - 6 \cdot 3 + 5 = -4$. Therefore, the vertex is: $(3, -4)$.

3) To find the y-intercept, we set x=0: $0^2 - 6 \cdot 0 + 5 = 5$.

4) To find the x-intercept(s), we set y=0: $x^2 - 6x + 5 = 0$. This factors into $(x-5)(x-1) = 0$. Therefore, $x = 5 \text{ or } x = 1$.

5)



Example 3:

Let $y = -2x^2 - 4x + 1$

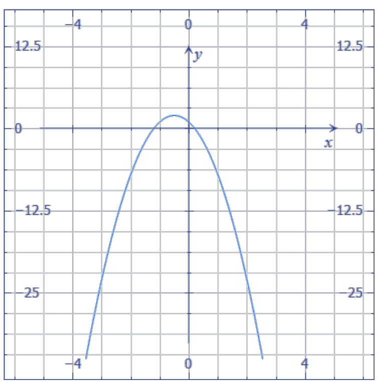
Let us graph this function using steps 1) – 6) above:

1) $a = -2$, which is negative. Therefore our parabola opens downward.

2) $x = -\frac{b}{2a} = \frac{4}{2\cdot(-2)} = -1.$ $y = -2(-4)^2 - 4\cdot(-1) + 1 = -27.$ Therefore, the vertex is: $(-1, -27)$.

3) To find the y-intercept, we set $x = 0$: $y = 1$.

4) To find the x-intercept(s), we set $y = 0$: $= -2x^2 - 4x + 1 = 0.$ This does not factor. We use the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{4\pm\sqrt{(-4)^2-4(-2)(1)}}{2(-2)} = \frac{4\pm\sqrt{16+8}}{-4} = \frac{4\pm\sqrt{24}}{-4} = \frac{4\pm2\sqrt{6}}{-4} = \frac{-2\pm\sqrt{6}}{2}$

5) 

Exercise Set 2:

For the following set of exercises, use the 5 steps listed above to graph the following functions:

1) $y = x^2 + 4x$
2) $y = x^2 - 5x + 6$
3) $y = x^2 + 7x + 10$
4) $y = x^2 + x - 6$
5) $y = 2x^2 + x - 3$

- Survey of Mathematics with Applications, Angel, Abbot, Runde, 8th ed.
  - Chapter 6.10.
- Mathematical Excursion...
  - Chapter 10.4